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Original Research Article

An Inventory Model for Delayed Deteriorating Items with Lead Time and Quadratic Time Dependent Holding Cost

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ABSTRACT
In this study, the inventory model for delayed deteriorating items with lead time, price dependent demand and Quadratic time dependent holding cost were developed. The proposed model assumes a price dependent demand before deterioration begins and constant demand rate at the onset of deterioration and beyond. Shortages are allowed and completely backlogged. The mathematical model developed is analytically solved and an iterative method is employed to find its numerical solution. Examples on the application of the model are provided and sensitivity analysis on some selected system parameters carried out.

INTRODUCTION

In recent years, inventory problem for deteriorating items have been widely studied. It is usually assume that a large quantity of goods in a stock will lead the customers to buy more goods. This situation motivates the retailer to increase their order quantity. But in real life situation, some type of items such as; fruits, food, vegetables, cakes, sweets and pharmaceuticals either deteriorates or become obsolete during store time.

Decay or deterioration of a product is a realistic situation associated with an inventory system. Approximately, all products deteriorate and loss their values partially or completely over time, when they are kept in stock as an inventory for fulfilling the future demand, which may occur due to one or many factors i.e. storage conditions, weather conditions or due to humidity. Therefore, the effect of deterioration is very important natural phenomenon. Ghare and Schrader (1963) was the pioneer to use the concept of deterioration. They developed an (Economic Order Quantity) EOQ model with a constant rate of decay. This inventory model laid foundations for the follow-up study in the inventory control for deteriorating products.

Abubakar and Babangida (2011) developed an inventory policy of delayed deteriorating items under permissible delay in payment. Abubakar and Babangida (2012), developed an EOQ model for delayed deteriorating items with linear time dependent holding cost. Abubakar and Samaila, (2017) Developed An inventory model for delayed deteriorating items with linear and time proportional demand under permissible delay in payment, Abubakar and Jamilu (2019), constructed an EOQ model for delayed deteriorating items with quadratic time dependent holding cost and backordering.

Lead time has been a topic of interest for many authors Ben-daya (1994), Das (1975), Foote (1988), Magson (1979), Naddor (1966), Chung and Ting (1993), Fujiwara (1993).

The decaying inventory problem was first analyzed by Ghare and Shrader (1963) who developed EOQ model with a constant rate of decay. This work was extended by Covert and Philip (1973) who developed EOQ model for a variable rate of deterioration. Abad (2001), with a profit par period objective, allows for flexible pricing.

Maragatham and Palani (2017) formulated a deterministic inventory model in which deterioration rate is time proportional, demand rate is function of selling price. M. Palanivel and S Gowri (2018) developed a production inventory model with delayed deteriorating items in which demand is a deterministic function of selling price.

Assumptions and notations

The following assumptions are considered in developing the model;

1. The demand rate at the time before deterioration sets in, is price depending and is of the form $D(p) = \alpha p^{-\beta}$, $\alpha, \beta > 0, p$ is selling price, "α" is initial demand.
2. The demand rate after deterioration sets in is constant; D_C
3. Replenishment rate is finite and the lead time is constant
4. Shortages are allowed
5. Holding cost is a function of time
6. Deterioration is not instantaneous

Notations;

The following notations are used in developing the model;

A_D: Amount of material deteriorated during a cycle time

$Q(t)$: Time dependent deterioration rate

C: Unit cost per item

A: the ordering cost

$D(p) = \alpha p^{-\beta}$, demand rate at time before deterioration sets in

D_C= Demand rate after deterioration sets in

t_i: Replenishment cycle time

L: Lead time

Pc; Purchase cost

Sc; Shortage cost

Ch; The total holding cost

Cd; Total deterioration cost per cycle

Q; Maximum inventory level

h; The inventory holding cost

$I_{0;}$The initial inventory

$I_1(t)$; The inventory level at any time before deterioration sets in

$I_2(t)$; The inventory level after deterioration sets in

μ; The rate of deterioration

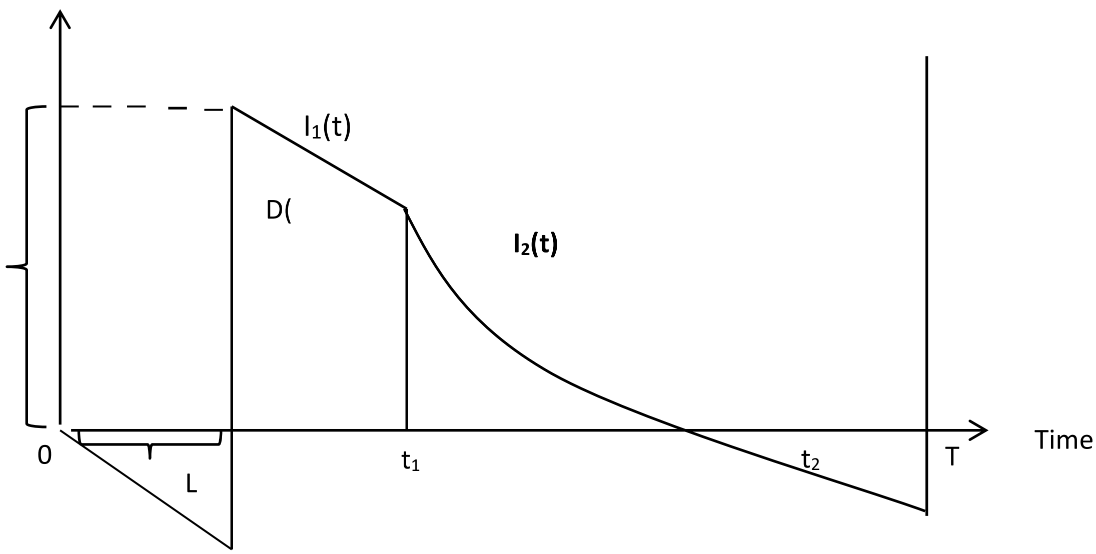

Figure 1: Inventory level

The Mathematical Model

The inventory level (Figure 1) in the interval $[L, t_1]$ before deterioration begins where depletion occurs only due to demand is described by the following;

$$\frac{d}{dt} I_1(t) = -\alpha p^{-\beta}, L \leq t \leq t_1 \quad (1)$$

Depletion of inventory in the interval [t_1 , t_2] after deterioration sets will be as a result of the combined effect of demand and deterioration which is represented by;

$$\frac{d}{dtI_2(t)} + \mu I_2(t) = Dc, t_1 \leq t \leq t_2 \quad (2)$$

Using the boundary conditions $I_1(L)$ = 0 and $I_2(t_2)$ = 0, the solutions of the above differential equations are

$$I_1(t) = \alpha p^{-\beta} (t_1 - t) - \frac{Dc}{\mu} (1 - e^{\mu(t2-t1)}) \quad (3)$$

$$Q = \alpha p^{-\beta} (t_1 - L) - \frac{Dc}{\mu} (1 - e^{\mu(t2-t1)}) \quad (4)$$

$$I_2(t) = \frac{Dc}{\mu} (e^{\mu(t_2-t)} - 1) \quad (5)$$

Now, cost due to deterioration of materials is computed as follows;

$$Cd = C(-\frac{Dc}{\mu}(1 - e^{\mu(t_2-t_1)}) + \mu(t_2 - t_1)) \quad (6)$$

The inventory holding cost is computed as follows;

Computation of the total inventory cost

The total inventory or variable cost is made up of the sum of the following;
Ordering cost A, the purchase cost pc which is given by;

$$Pc = c[Q + LDp] = c[\alpha p^{-\beta} t1 - \frac{Dc}{\mu}(1 - e^{\mu(t2-t1)})] \quad (7)$$

The deterioration cost dc, which is computed as;

$$Dc = C(\frac{Dc}{\mu}(e^{\mu(t2-t1)} - 1) + \mu(t2 - t1)) \quad (8)$$

$$The\ shortage\ cost\ Sc = s\,Dc\frac{(T-t_2)^2}{2} \quad (9)$$

And the holding cost given by;

$$CH = IC\int_L^{t_1}(a+bt+ct^2)((\alpha p^{-\beta}(t_1 - t) - \frac{Dc}{\mu}(1 - e^{\mu(t_2-t_1)})) + IC\int_{t_1}^{t_2}\left[(a+bt+ct^2)\frac{Dc}{\mu}(e^{\mu(t_2-t)} - 1)\right]dt$$

$$= \mathrm{ic}(a\alpha p^{-\beta}t_1(t_1 - L) - \frac{a\alpha p^{-\beta}}{2}(t_1^2 - L^2) - a\frac{Dc}{\mu}(t_1 - L) + a\frac{Dc}{\mu}e^{\mu(t_2-t_1)}(t_1 - L) + b\frac{\alpha p^{-\beta}t_1}{2}(t_1^2 - L^2) - b\frac{\alpha p^{-\beta}t_1}{3}(t_1^3 - L^3) - b\frac{Dc}{2\mu}(t_1^2 - L^2) + b\frac{Dc}{2\mu}e^{\mu(t_2-t_1)} + c\frac{\alpha p^{-\beta}t_1}{3}(t_1^3 - L^3) - c\frac{\alpha p^{-\beta}}{4}(t_1^4 - L^4) - c\frac{Dc}{3\mu}e^{\mu(t_2-t_1)}(t_1^3 - L^3) - a\frac{Dc}{\mu^2}(1 - e^{\mu(t_2-t_1)}) - a\frac{Dc}{\mu}(t_2 - t_1) - b\frac{Dc}{\mu^2}\left((t_2 - t_1)(1 - e^{\mu(t_2-t_1)}) + \frac{1}{\mu}(1 - e^{\mu(t_2-t_1)})\right) - b\frac{Dc}{2\mu}(t_2^2 - t_1^2) + c\frac{Dc}{\mu}\left(\left(\frac{-1}{\mu}(t_2^2 - t_1^2)(1 - e^{\mu(t_2-t_1)})\frac{-2}{\mu}(t_2 - t_1) - \frac{2}{\mu^3}(1 - e^{\mu(t_2-t_1)})\right) - c\frac{Dc}{3\mu}(t_2^3 - t_1^3)\right)) \quad (10)$$

The total inventory or variable cost is made up of the sum of the following;
Ordering cost A, the purchase cost p_c, cost due to deterioration and shortage cost which are given by;

$$TIC(T) = \frac{1}{T} + [A + Pc + dc + Ch + Sc]$$

$$= \frac{A}{T} + \frac{iC}{T}(a\alpha p^{-\beta}t_1(t_1 - L) - \frac{a\alpha p^{-\beta}}{2}(t_1^2 - L^2) - a\frac{Dc}{\mu}(t_1 - L) + a\frac{Dc}{\mu}e^{\mu(t_2-t_1)}(t_1 - L) + b\frac{\alpha p^{-\beta}t_1}{2}(t_1^2 - L^2) - b\frac{\alpha p^{-\beta}t_1}{3}(t_1^3 - L^3) - b\frac{Dc}{2\mu}(t_1^2 - L^2) + b\frac{Dc}{2\mu}e^{\mu(t_2-t_1)} + c\frac{\alpha p^{-\beta}t_1}{3}(t_1^3 - L^3) - c\frac{\alpha p^{-\beta}}{4}(t_1^4 - L^4) - c\frac{Dc}{3\mu}e^{\mu(t_2-t_1)}(t_1^3 - L^3) - a\frac{Dc}{\mu^2}(1 - e^{\mu(t_2-t_1)}) - a\frac{Dc}{\mu}(t_2 - t_1) - b\frac{Dc}{\mu^2}\left((t_2 - t_1)(1 - e^{\mu(t_2-t_1)}) + \frac{1}{\mu}(1 - e^{\mu(t_2-t_1)})\right) - b\frac{Dc}{2\mu}(t_2^2 - t_1^2) + c\frac{Dc}{\mu}\left(\left(\frac{-1}{\mu}(t_2^2 - t_1^2)(1 - e^{\mu(t_2-t_1)})\frac{-2}{\mu}(t_2 - t_1) - \frac{2}{\mu^3}(1 - e^{\mu(t_2-t_1)})\right) - c\frac{Dc}{3\mu}(t_2^3 - t_1^3)\right)) + \frac{C}{T}[\alpha p^{-\beta}t1 - \frac{Dc}{\mu}(1 - e^{\mu(t_2-t_1)})] + \frac{C}{T}(\frac{Dc}{\mu}(e^{\mu(t_2-t_1)} - 1) + \mu(t_2 - t_1)) + \frac{s\,Dc}{T}\frac{(T-t_2)^2}{2} \quad (11)$$

The objective is to minimize the total variable cost and optimize the profit. We differentiate (11) with respect to multiplying with T^2 and equate to zero to obtain the value of the cycle length T which gives the minimum total variable (inventory) cost as follows;

$$\frac{dTC(T)}{dT}T^2 = -A - ic(a\alpha p^{-\beta}t_1(t_1 - L) - \frac{a\alpha p^{-\beta}}{2}(t_1^2 - L^2) - a\frac{Dc}{\mu}(t_1 - L) + a\frac{Dc}{\mu}e^{\mu(t_2-t_1)}(t_1 - L) + b\frac{\alpha p^{-\beta}t_1}{2}(t_1^2 - L^2) - b\frac{\alpha p^{-\beta}t_1}{3}(t_1^3 - L^3) - b\frac{Dc}{2\mu}(t_1^2 - L^2) + b\frac{Dc}{2\mu}e^{\mu(t_2-t_1)} + c\frac{\alpha p^{-\beta}t_1}{3}(t_1^3 - L^3) - c\frac{\alpha p^{-\beta}}{4}(t_1^4 - L^4) - c\frac{Dc}{3\mu}e^{\mu(t_2-t_1)}(t_1^3 - L^3) - a\frac{Dc}{\mu^2}(1 - e^{\mu(t_2-t_1)}) - a\frac{Dc}{\mu}(t_2 - t_1) - b\frac{Dc}{\mu^2}\left((t_2 - t_1)(1 - e^{\mu(t_2-t_1)}) + \frac{1}{\mu}(1 - e^{\mu(t_2-t_1)})\right) - b\frac{Dc}{2\mu}(t_2^2 - t_1^2) + c\frac{Dc}{\mu}\left(\left(\frac{-1}{\mu}(t_2^2 - t_1^2)(1 - e^{\mu(t_2-t_1)})\frac{-2}{\mu}(t_2 - t_1) - \frac{2}{\mu^3}(1 - e^{\mu(t_2-t_1)})\right) - c\frac{Dc}{3\mu}(t_2^3 - t_1^3)\right)) - c[\alpha p^{-\beta}t_1 - \frac{Dc}{\mu}(1 - e^{\mu(t_2-t_1)})] - C(\frac{Dc}{\mu}(e^{\mu(t_2-t_1)} - 1) + \mu(t_2 - t_1)) + s\frac{Dc}{2}(T^2 - t_2^2) = 0 \quad (12)$$

Equation (12) is used to determine the best cycle length T which minimizes the total inventory cost. $\frac{d^2I(t)}{dT^2}$ was found to be greater than zero.

Solution Algorithms
Step 1, Start
Step 2, Impute the appropriate values of the system parameters
Step 3, Using Maple 18 software solve equation (12) to find the value of T
Step 4, Keep repeating step 3 until you get the optimal value of T
Step5, Substitute the value of T in equation 13 to obtain the optimal total inventory cost TVC(T)
Step 6, End.

Numerical Example
For the numerical example, the values of various parameters in proper units can be taken as follows:
A=200, $\alpha = 100, \beta = 0.2, a = 0.1, C = 15, b = 0.02, c = 8, p = 20, Dc = 50, i = 0.06, s = 100, L = 0.0192, t_1 = 0.0192,\ t_2 = 0.0384,\ \mu = 0.024.$
Solving equation (12) with the above parameter values, we obtain the cycle length T=0.3. On substituting the value of T in (11), we obtain the minimum total cost TC=7907.

Table 1: Effect of changes in the values of some system parameters

S/N	A(#)	α(Units)	μ	a	B	C	p	Dc	s	L	C	T	Tvc
1	200	100	0.02	0.1	0.2	15	20	50	100	0.192	8	0.33	14,482
2	250	200	0.11	0.2	0.1	20	30	45	120	0.172	5	0.122	55,438
3	300	300	0.77	0.3	0.8	30	29	38	150	0.183	3	0.191	64,311
4	600	250	0.43	0.03	0.91	45	43	27	180	0.213	11	0.261	44,831
5	400	285	0.28	0.36	0.72	50	48	30	115	0.012	13	0.352	53,312

CONCLUSION
The inventory model for delayed deteriorating items with lead time and quadratic time dependent holding cost in which the demand rate before deterioration sets in is price dependent, and after deterioration begins is assumed to be constant were developed. Shortages are allowed and completely backlogged in the present. Finally, five numerical examples are given to explain the solution algorithm.

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