



Hybrid Numerical Solution of Sequential Impulsive Pursuit and Evasion Games in Cislunar Orbital Dynamics

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ABSTRACT

Impulsive orbital pursuit–evasion in cislunar space presents computational challenges arising from nonlinear Earth–Moon gravitational interactions and discrete manoeuvre constraints. This study formulates a sequential zero-sum minimum-time interception game using the circular restricted three-body problem (CR3BP). Pontryagin's maximum principle provides necessary optimality conditions expressed as a multipoint boundary-value problem (MPBVP). A hybrid initialization procedure combines genetic-algorithm exploration, coarse-grid time search and differential correction for single-impulse manoeuvres, while a receding-horizon procedure supplies initial estimates for multiple-impulse cases. The contribution lies in integrating nonlinear cislunar propagation, observed evader actions, constrained pursuer responses and boundary-value refinement into a unified computational framework. Numerical examples involving distant retrograde orbits investigate departure phase, initial orbital configuration, manoeuvre timing and impulse capability. In the reported co-orbital benchmark, fixed-angle enumeration with 30° direction increments and 100/T time steps required 51.9305 minutes, compared with 5.3737 minutes for the hybrid approach. These benchmark-specific results demonstrate computational potential within the tested configurations; the computed trajectories remain candidate solutions and do not establish a globally optimal Stackelberg equilibrium.

CITATION

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INTRODUCTION

Cislunar space is an increasingly important operational environment because coupled Earth–Moon gravitational dynamics admit trajectories that cannot be represented adequately by simple two-body approximations. Earth–Moon transfer corridors, libration-point neighbourhoods and periodic orbital families offer dynamically structured pathways relevant to future spacecraft operations. As lunar exploration activities expand, reliable prediction and control in this environment become increasingly important (Almeida et al., 2025; Kumar et al., 2025). Studies of cislunar periodic solutions further illustrate the

significance of the underlying nonlinear orbital geometry (Atallah & Servadio, 2024).

Increasing activity in the lunar vicinity creates challenges for proximity operations, orbital coordination and the analysis of competing manoeuvres. Navigation, situational awareness and governance have accordingly gained attention in the cislunar literature (Paladini, 2026). Although adversarial spacecraft scenarios should not be interpreted as operational forecasts, they provide a mathematical setting in which to examine strategic responses and orbital safety (Elizabeth, 2025; Mehlman et al., 2025). In an orbital pursuit–evasion game, the pursuer

seeks interception while the evader seeks to prevent or delay it (Li et al., 2025).

Differential-game theory describes such interactions through competing objectives, admissible controls and information-dependent decision rules (Weintraub et al., 2020; Yang & Gao, 2013; Zeeuw, 2024). Its application to spacecraft motion requires a distinction between the physical dynamics and the strategic solution concept. In particular, necessary optimality conditions and their numerical satisfaction do not, without further analysis, establish a globally optimal game equilibrium (Xu et al., 2025).

Earlier orbital pursuit–evasion research has addressed three-dimensional relative motion and the solution of constrained interception problems (Fu et al., 2024; Pontani & Conway, 2009). Impulsive Stackelberg formulations explicitly represent the order in which an evader acts and a pursuer responds (Venigalla & Scheeres, 2021; Fu et al., 2025). However, strategies developed for Keplerian reference motion cannot automatically be assumed to capture the strongly nonlinear dynamics of the Earth–Moon CR3BP. The present work therefore treats sequential decisions and cislunar propagation together, rather than considering the evader as a passive target.

Related numerical approaches include local Hill-frame trajectory representations, nonlinear feedback guidance and computational studies of manoeuvring conflicts (Starek et al., 2016; Bardhan & Ghose, 2015; Wei et al., 2023). Hierarchical optimisation has also been used to separate high-level decisions from trajectory refinement (He et al., 2022; Gao et al., 2025). These methods provide relevant computational ingredients, but they do not by themselves supply the particular combination of Earth–Moon CR3BP dynamics, sequential impulse decisions and hybrid initialization examined here.

The assumed propulsion model and information structure materially affect the game. Continuous-thrust and impulsive-control formulations impose different control constraints (Hafer et al., 2015), while manoeuvre timing and available observations influence the decisions that can be made (Ye et al., 2020). Moreover, sequential leader–follower interaction and simultaneous-move games have different information patterns and need not admit interchangeable equilibrium interpretations (Gupta et al., 2025). Consequently, the minimum-time sequential game considered here is distinguished from fixed-terminal-time, terminal-separation comparisons.

Learning-based numerical policies have been explored for collision avoidance, intermittent pursuit and impulsive non-cooperative manoeuvring (Ferrara et al., 2025; Liu et al., 2025; Yan et al., 2024). Their performance nevertheless depends on training assumptions and the scenarios sampled. Such methods complement rather than replace a boundary-value formulation when the objective is to examine necessary conditions and constrained candidate

interception trajectories under a specified dynamical model.

Cislunar pursuit–evasion is especially sensitive to gravitational geometry and manoeuvring authority (Halsey et al., 2024; Han & Dang, 2023). Studies of the Earth–Moon L1 vicinity, Hamilton–Jacobi–Isaacs approaches and reinforcement-learning guidance illustrate both the dynamical interest of this setting and its computational difficulty (Fotiadis et al., 2025; Jagat & Sinclair, 2017; Yan et al., 2019; Zhang et al., 2024; LaFarge et al., 2020). These studies motivate careful numerical initialization without implying that solutions obtained under one orbital or information regime are globally valid in another.

Discrete manoeuvres are particularly pertinent when spacecraft have finite velocity-change budgets. Reachability-based research and non-cooperative cislunar inspection studies illustrate the importance of attainable trajectories and strategic interaction (Bowerfind & Taheri, 2024; Fu et al., 2025). Research on libration-point mission design and nonlinear Earth–Moon transfers also shows why accurate propagation and well-chosen initial guesses are essential in this setting (Ross et al., 2004; Campana et al., 2024). The unresolved practical issue addressed by this study is how to assemble these established modelling and numerical elements into a tractable sequential impulsive interception workflow.

The distinctive contribution of the present study is an integrated CR3BP-based sequential impulsive pursuit–evasion formulation coupled to a hybrid numerical initialization strategy. The model connects the evader-first observation–response structure, bounded instantaneous velocity changes and a minimum-capture-time objective with Pontryagin-based multipoint boundary conditions. A genetic-algorithm search over evader impulse directions is combined with coarse-to-fine pursuer timing, differential correction and shooting refinement; a receding-horizon construction provides staged initial guesses for repeated manoeuvres. These components are not claimed individually as new theories or algorithms. The aim of this study is to formulate and numerically examine candidate minimum-time solutions for sequential impulsive pursuit–evasion in Earth–Moon CR3BP dynamics using this integrated hybrid approach.

Game setting, state variables and manoeuvre epochs

Consider two spacecraft governed by Earth Moon circular restricted three-body dynamics: a pursuer, P , seeking interception and an evader, E , seeking to delay interception. Their pre-manoevrue paths are prescribed. The interaction is sequential: the evader manoeuvres first, whereas the pursuer acts after the prescribed observation delay. Both spacecraft undergo instantaneous velocity changes without position discontinuities. Let $i \in \{P, E\}$ Denote spacecraft identity and $j \in \{1, \dots, n\}$ the manoeuvre round; the spacecraft state is (q_i, u_i) ,

comprising the rotating-frame position and velocity vectors.

For a periodic reference trajectory, the phase is described by its initial value and the reference angular rate. The starting epoch is explicitly subtracted so that the phase relation remains valid for arbitrary time origins:

$$\varphi(t) = \varphi_0 + n_{\text{orb}}(t - t_s), \quad n_{\text{orb}} = \frac{2\pi}{P_{\text{orb}}} \quad (1)$$

Figure 1. Orbital pursuit–evasion scenario

Let t_s be the first evader manoeuvre epoch and $\delta_{\text{obs}} \geq 0$ the observation delay. A fixed inter-manoeuve interval Δt_m gives the following epoch schedules:

$$\begin{cases} s_{E,j} = t_s + (j-1)\Delta t_m \\ s_{P,j} = t_s + \delta_{\text{obs}} + (j-1)\Delta t_m \end{cases} \quad j = 1, \dots, n_m \quad (2)$$

Each player has a finite instantaneous velocity-change capability. For every admissible manoeuvre, the Euclidean magnitude of its velocity increment is bounded by:

$$\|\delta u_i\| \leq \bar{u}_{\text{imp},i}, \quad i \in \{P, E\} \quad (3)$$

The state-update rule is imposed at each manoeuvre epoch $s\{i, j\}$, where superscripts $-$ and $+$ denote the left and right limits, respectively:

$$\begin{cases} q_i(s_{i,j}^+) = q_i(s_{i,j}^-) \\ u_i(s_{i,j}^+) - u_i(s_{i,j}^-) = \delta u_{i,j} \end{cases} \quad (4)$$

$$q_P(t_c) = q_E(t_c) \quad (5)$$

The capture epoch t_c is measured from the same time origin as the initial manoeuvre epoch t_s on the observed evader action, the pursuer seeks the earliest feasible interception, while the evader seeks to delay the resulting response. This order of decisions motivates the nested Stackelberg formulation; by itself, however, it does not establish existence, uniqueness or global optimality of an equilibrium.

Rotating-Frame Dynamics and Non-dimensionalization

The CR3BP assumes circular motion of the two primaries around their common barycentre and negligible spacecraft mass. The rotating X axis points from Earth toward Moon, the Z axis is normal to the primaries' orbital plane, and Y completes the right-handed frame. With D_{EM} as characteristic Earth–Moon separation and GM_E and GM_M as dimensional gravitational parameters, the characteristic scales are:

$$L^* = D_{EM}, \quad T^* = \sqrt{\frac{D_{EM}^3}{GM_E + GM_M}} \quad (6)$$

$$\nu = \frac{GM_M}{GM_E + GM_M} \quad (7)$$

Using normalized rotating-frame positions $q_i = (X_i, Y_i, Z_i)^T$ and velocities $u_i = dq_i/dt$, the CR3BP acceleration $a_i = d^2q_i/dt^2$ can be expressed componentwise as follows:

$$a_i = \begin{bmatrix} 2\dot{Y}_i + X_i - (1-\nu)\frac{X_i+\nu}{\rho_{1i}^3} - \nu\frac{X_i-1+\nu}{\rho_{2i}^3} \\ -2\dot{X}_i + Y_i - (1-\nu)\frac{Y_i}{\rho_{1i}^3} - \nu\frac{Y_i}{\rho_{2i}^3} \\ -(1-\nu)\frac{Z_i}{\rho_{1i}^3} - \nu\frac{Z_i}{\rho_{2i}^3} \end{bmatrix} \quad (8)$$

In this representation, the spacecraft distances to Earth and Moon, respectively, are scalar quantities rather than position vectors:

$$\rho_{1i}^2 = (X_i + \nu)^2 + Y_i^2 + Z_i^2, \quad \rho_{2i}^2 = (X_i - 1 + \nu)^2 + Y_i^2 + Z_i^2 \quad (9)$$

Optimality Conditions and Differential Game Formulation

Performance criteria and Hamiltonian

For the sequential minimum-time interception problem, the performance functional is the capture epoch defined by the following expression:

$$J = t_c \quad (10)$$

For a prescribed terminal time, the terminal position separation may instead be used as an auxiliary reachable-set criterion. This auxiliary criterion is distinct from the minimum-time game objective:

$$J_d = \|q_P(t_c) - q_E(t_c)\| \quad (11)$$

Introduce adjoints $p_{\{q,i\}}$ and $p_{\{u,i\}}$ corresponding to q_i and u_i . The instantaneous Hamiltonian for each free-flight arc is the dual pairing of the costates with the CR3BP state dynamics:

$$\mathcal{H}_i = p_{q,i}^T u_i + p_{u,i}^T a_i \quad (12)$$

Differentiating with respect to q_i and u_i yields the costate dynamics. As the acceleration is independent of velocity except for rotating-frame Coriolis coupling, that coupling must be accounted for in the full state Jacobian; the corrected full costate equation includes this velocity Jacobian, unlike the abbreviated source expression:

$$\dot{p}_{q,i} = -\left(\frac{\partial a_i}{\partial q_i}\right)^T p_{u,i}, \quad \dot{p}_{u,i} = -p_{q,i} - \left(\frac{\partial a_i}{\partial u_i}\right)^T p_{u,i} \quad (13)$$

For compact matrix notation, let $d\{1i\} = (X_i + \nu, Y_i, Z_i)^T$ and $d\{2i\} = (X_i - 1 + \nu, Y_i, Z_i)^T$. The position derivative of acceleration is:

$$\frac{\partial a_i}{\partial q_i} = \text{diag}(1, 1, 0) - (1-\nu) \left(\frac{I_3}{\rho_{1i}^3} - \frac{3d_{1i}d_{1i}^T}{\rho_{1i}^5} \right) - \nu \left(\frac{I_3}{\rho_{2i}^3} - \frac{3d_{2i}d_{2i}^T}{\rho_{2i}^5} \right) \quad (14)$$

Terminal and impulsive boundary conditions

With free capture time and the interception constraint $q_P(t_c) = q_E(t_c)$, the augmented terminal expression is:

$$\Psi_f = t_c + \xi^T (q_P(t_c) - q_E(t_c)) \quad (15)$$

The terminal multiplier ξ couples the two positions. The corresponding position adjoints satisfy:

$$p_{q,P}(t_c) = \xi, \quad p_{q,E}(t_c) = -\xi \quad (16)$$

Therefore their sum vanishes at capture:

$$p_{q,P}(t_c) + p_{q,E}(t_c) = 0 \quad (17)$$

Because terminal velocities are unconstrained, the velocity adjoints satisfy:

$$p_{u,P}(t_c) = 0, \quad p_{u,E}(t_c) = 0 \quad (18)$$

For free capture time, the transversality condition is stated using the Hamiltonian sign convention adopted in the formulation:

$$\mathcal{H}(t_c) = -\frac{\partial \Psi_f}{\partial t_c} = -1 \quad (19)$$

At a pursuer impulse, a magnitude-bound constraint introduces the multiplier $\gamma_{imp,P}$:

$$\Psi_{imp} = \gamma_{imp,P} (\|\delta u_P\| - \bar{u}_{imp,P}) \quad (20)$$

At an interior manoeuvre epoch, the stated matching conditions retain continuity of the position costate and describe the prescribed velocity-costate jump:

$$p_{q,P}(s_P^-) = p_{q,P}(s_P^+), \quad p_{u,P}(s_P^-) = p_{u,P}(s_P^+) - \gamma_{imp,P} \frac{\delta u_P}{\|\delta u_P\|} \quad (21)$$

At the initial epoch the evader's velocity is impulsively changed while the pursuer initially retains its pre-manoeuver velocity:

$$u_E(t_s^+) = u_E(t_s^-) + \delta u_E, \quad u_P(t_s^+) = u_P(t_s^-) \quad (22)$$

Under the source's signed-multiplier convention, complementary slackness is expressed below; the admissibility inequalities $\|\delta u_i\| \leq \bar{u}_{imp,i}$ remain in force for both players:

$$\begin{aligned} \gamma_{imp,P} (\|\delta u_P\| - \bar{u}_{imp,P}) &= 0, \quad \gamma_{imp,P} > 0 \\ \gamma_{imp,E} (\|\delta u_E\| - \bar{u}_{imp,E}) &= 0, \quad \gamma_{imp,E} < 0 \end{aligned} \quad (23)$$

Together, the terminal conditions, impulse-matching relations and inequality constraints define the necessary conditions of the impulsive multipoint boundary-value problem. They link discrete manoeuvre decisions to free-flight CR3BP arcs. Numerical satisfaction of these conditions alone cannot establish a unique or globally optimal Stackelberg equilibrium.

REVIEW METHODOLOGY

Nested Strategy Optimization and Initialization

The evader selects a manoeuvre in its feasible strategy set $\mathcal{A}_E$; the pursuer then minimizes capture time over $\mathcal{A}_P$, conditional on that choice. With this decision order, the sequential equilibrium objective is:

$$t_c^* = \max_{\delta u_E \in \mathcal{A}_E} \min_{\delta u_P \in \mathcal{A}_P} t_c(\delta u_E, \delta u_P) \quad (24)$$

The outer genetic-algorithm search explores feasible evader impulse directions without exhaustive fine-grid enumeration. Conditional on each evader candidate, the inner pursuer-response procedure uses a coarse time search to identify feasible interception intervals,

differential correction to reduce terminal position mismatch and local refinement to obtain a candidate minimum interception time. The resulting estimates initialize the multipoint shooting calculation. The methodological contribution is the integration of these stages within the stated sequential CR3BP game, not the invention of each constituent algorithm. Numerical convergence provides a candidate solution rather than an independent guarantee of global optimality.

Differential Correction and Multiple Manoeuvres

Let e_q denote the terminal pursuer-minus-evader position residual at a trial flight time, and let $S_{qu}(t)$ be the position sensitivity to an initial velocity variation. The linearized correction is:

$$\Delta u_0^{\text{corr}} = -S_{qu}(t)^{-1} e_q, \quad \delta u_{P,0}^{(k+1)} = \delta u_{P,0}^{(k)} + k_{bc} \Delta u_0^{\text{corr}} \quad (25)$$

The stated iteration continues until the terminal position-residual norm falls below the specified tolerance ε_q , after which the impulse-magnitude constraint is assessed. The sensitivity-based correction is retained as given in the original formulation; its invertibility conditions and any near-singularity fallback remain matters for author confirmation.

For more than one manoeuvre round, receding-horizon initialization solves successive single-impulse subproblems. Where interception is not obtained in the current interval, both spacecraft states are propagated to the following manoeuvre epoch. The accumulated impulses and costate estimates then initialize multipoint shooting refinement. This sequential construction is a practical initialization device and is not, in general, equivalent to a globally optimal multistage strategy.

RESULTS AND DISCUSSION

Departure-phase dependence

The departure-phase experiment suppresses evader manoeuvring and varies the pursuer's starting phase along a common distant retrograde orbit (DRO). The selected reference orbit has amplitude 107,950 km and a period of 20.7 days. With the pursuer impulse capped at 50 m/s, the attainable interception patterns depend strongly on initial phase. The source distinguishes outward transfer, orbital loitering and lunar gravity assisted interception branches, together with phases for which the prescribed solver does not produce a feasible interception.

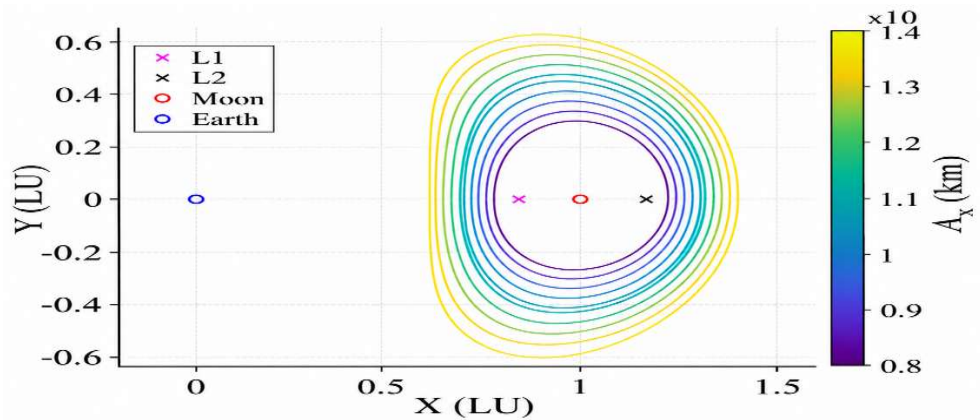


Figure 2: DRO orbit family

Table 1: DRO parameters

State	Evader
x	0.70728467
y	0
z	0
v_x	0
v_y	0.67893531
v_z	0

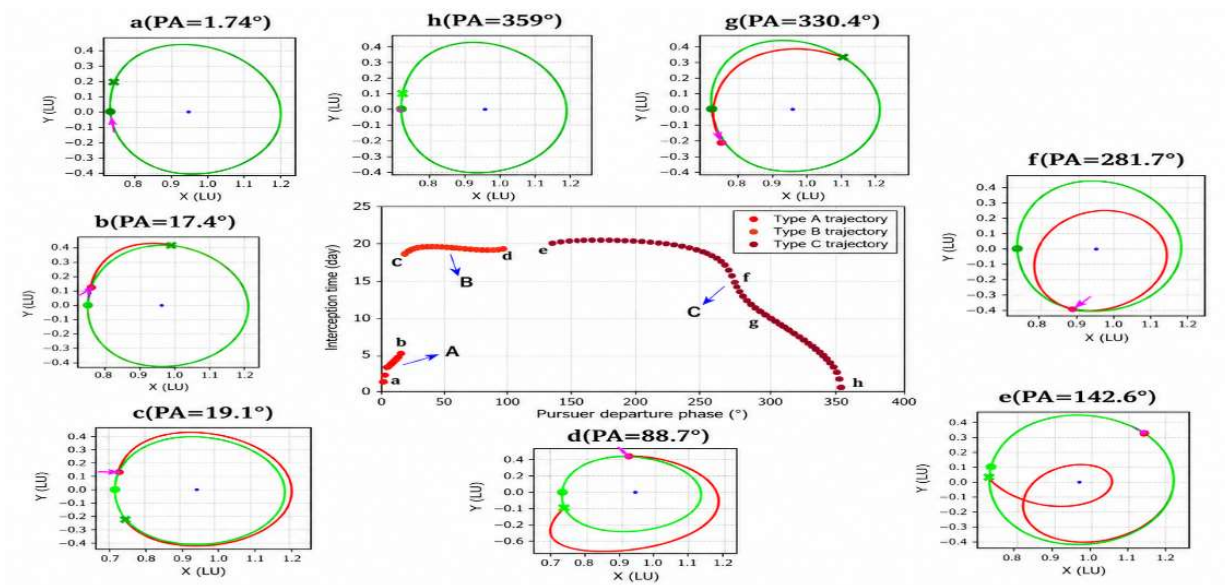


Figure 3: Pursuer interception trajectory types for different departure phases

Co-orbital minimum-time game

For the co-orbital case, the pursuer begins at phase 8.7°, with maximum velocity impulses of 50 m/s for P and 20 m/s

for E. Reachable-set calculations at fixed terminal times of 3, 4 and 5 days illustrate the transition from disjoint reachable regions to coverage of the evader region.

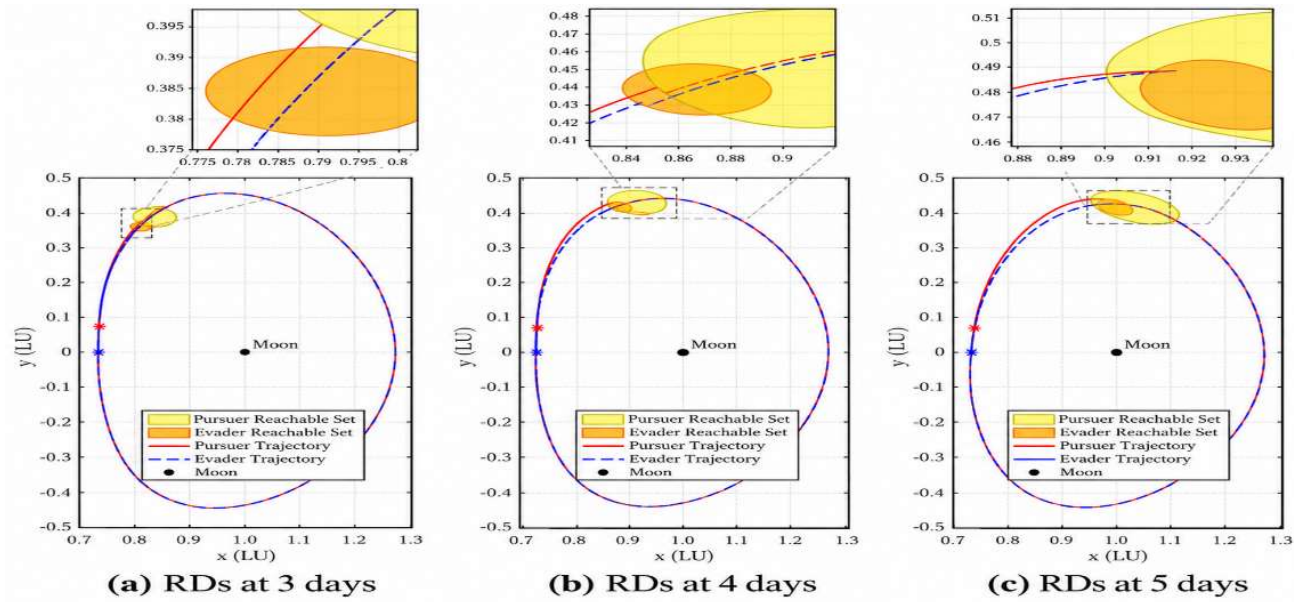


Figure 4: RDs in the co-orbital DRO pursuit-evasion game

The initialization uses the genetic-algorithm settings in Table 2 and differential-correction tolerance 10^{-8} , with sequential quadratic programming for local refinement.

Table 2: Algorithm Parameters for the Initialization Strategy

Parameter	Value
Population size	24
Max generations	18
Function tolerance	10^{-4}
Crossover type	Simulated binary

The resulting reported minimum-time interception is 4.5529 days. The accompanying solution describes nearly tangent reachable boundaries at the computed solution

and numerical convergence of the shooting iteration. Its static fixed-time comparison gives a Nash-game outcome different from the sequential Stackelberg-game result.

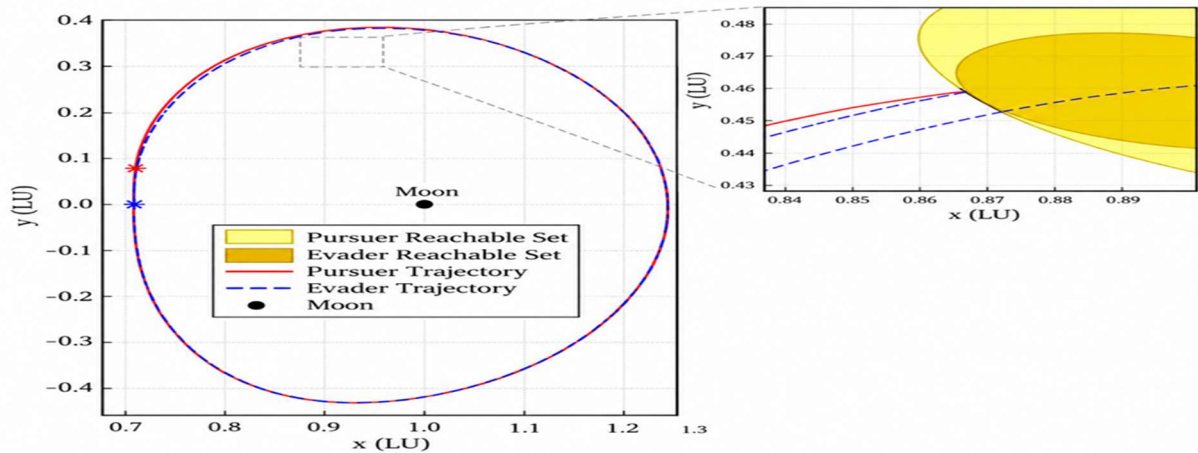


Figure 5: Pursuit-evasion analysis on the DRO

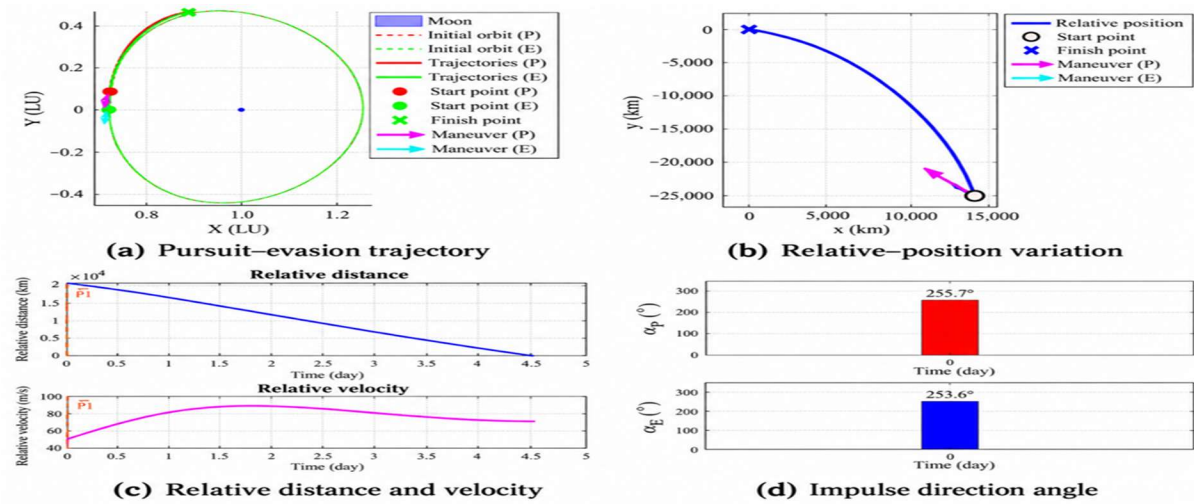


Figure 6. RDs in the co-orbital DRO pursuit-evasion game

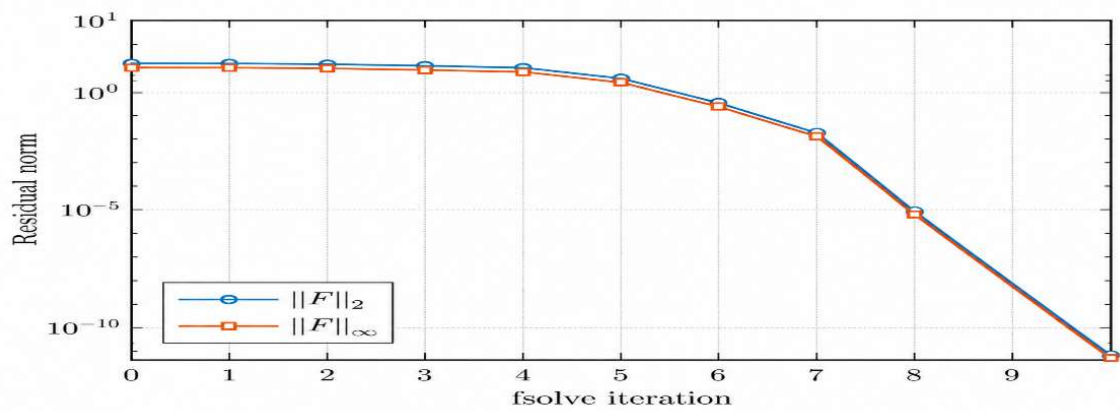


Figure 7: Convergence of the shooting method for the co-orbital DRO scenario

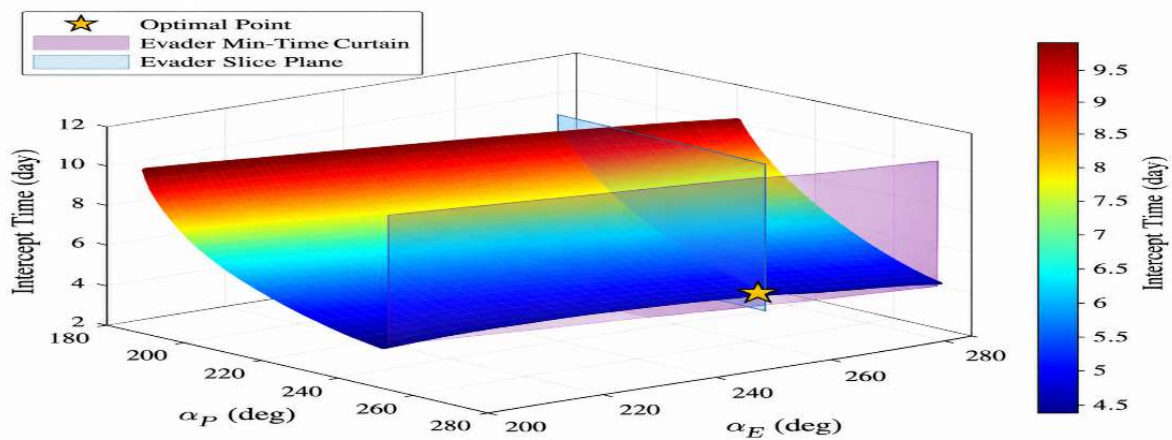


Figure 8: Scatter plot of interception time and direction angles of the two players for different impulse directions

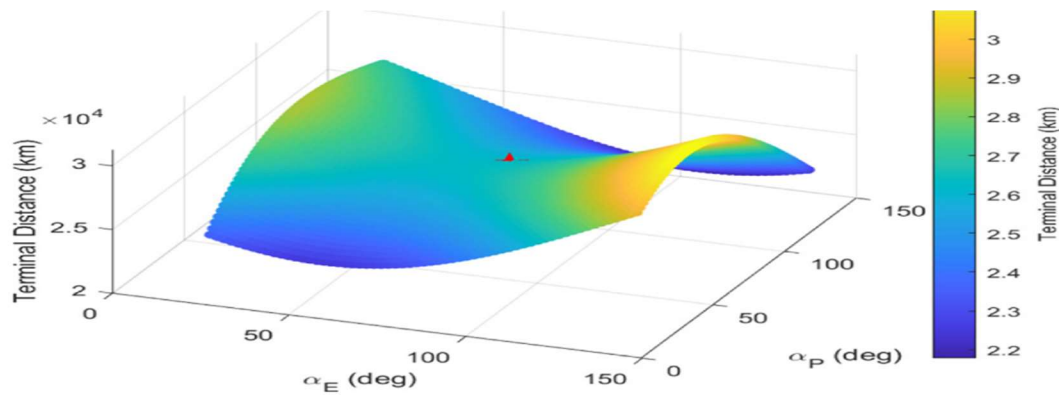


Figure 9: Nash equilibrium solution of the static game with fixed terminal time and terminal distance as the performance objective

The shooting-method convergence plot (original Figure 8; renumbered Figure 7) shows decreasing residual norms across iterations, indicating convergence of the displayed numerical candidate under the illustrated conditions; it does not establish global optimality.

The interception-time surface (original Figure 9; renumbered Figure 8) illustrates how the computed capture time varies with the pursuer and evader impulse-direction angles. The highlighted point is a numerical candidate within the plotted search region rather than proof of a global optimum.

The static-game surface (original Figure 10; renumbered Figure 9) displays terminal separation as a function of

manoeuvre-direction choices for a fixed terminal time. Its Nash interpretation differs from the sequential minimum-time formulation in both decision structure and objective, so the outcomes are not directly interchangeable.

Interception from distinct DROs

For distinct initial DROs, Table 3 gives the two dimensionless spacecraft states. The pursuer and evader velocity-change limits are 75 and 68 m/s, respectively. The reported interception time is 15.5334 days. Both trajectories exploit lunar gravity assisted motion; the relative distance varies non-monotonically before the reported terminal interception.

Table 3: Initial States of the Two Players in Different DROs

State	Pursuer	Evader
x	0.71559755	0.75047039
y	0.15223642	0
z	0	0
V_x	0.07287771	0
V_y	0.62693739	0.59959427
V_z	0	0

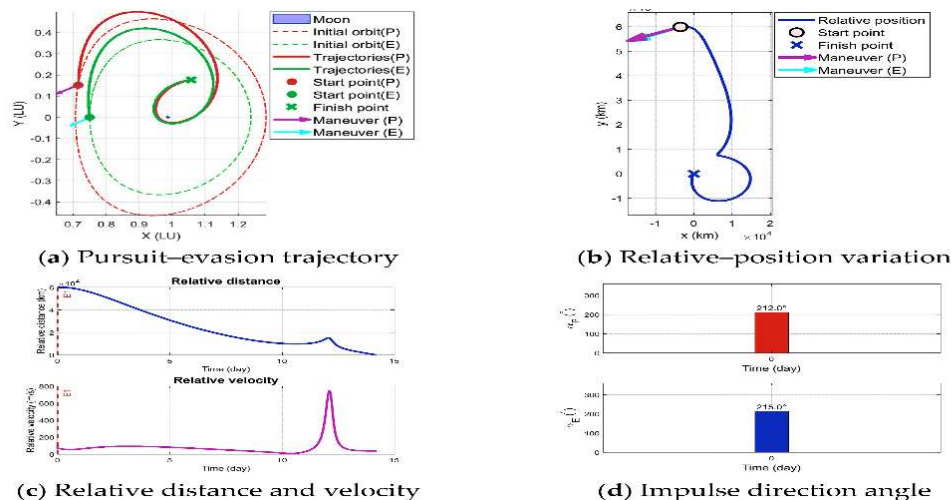


Figure 10: Analysis of DRO pursuit-evasion with lunar gravity assisted flight

Impulsive manoeuvres

The co-orbital multiple-round case applies impulse limits of 50 m/s for P and 30 m/s for E with $\Delta t_m = 2$ days. Three manoeuvre rounds produce a capture time of 4.4582 days. In the distinct-DRO case, respective limits of 70 and 40 m/s

and the same spacing yield four manoeuvre rounds and capture after 6.5457 days. In both cases, discontinuous relative-velocity changes coincide with the impulse epochs.

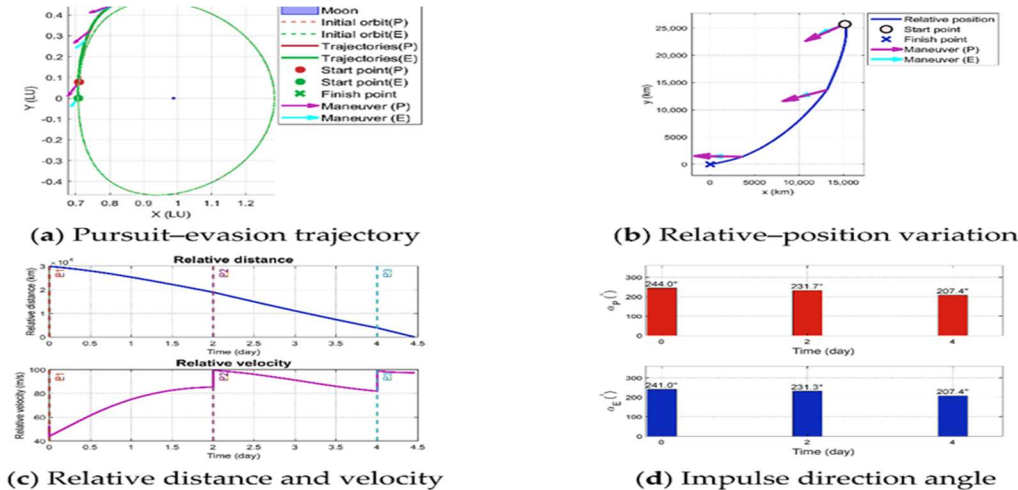


Figure 11: Analysis of co-orbital DRO multiple-round impulses

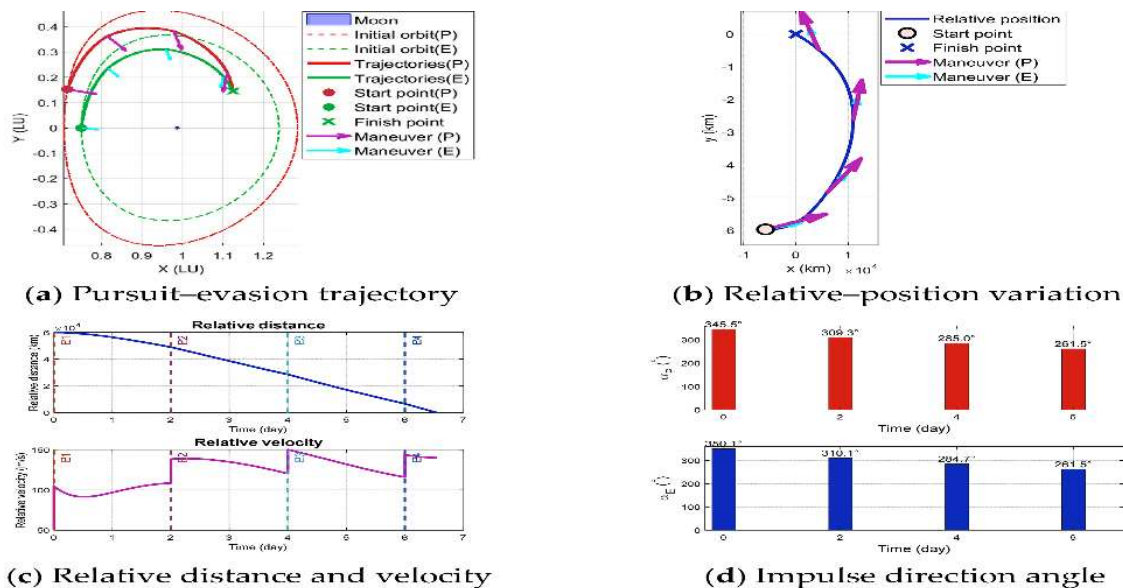


Figure 12: Analysis of different-orbit DRO multiple-round impulses

The impulse-spacing experiment, summarized in Table 4, shows progressively longer interception times as the

manoeuvre interval increases from one to four days while the count of manoeuvre rounds decreases.

Table 4: Effect of Impulse Interval on Interception Time and Impulse Rounds in the Co-Orbital DRO Scenario

Impulse Interval T (Days)	Interception Time (Days)	Number of Impulse Rounds
1	3.6314	4
2	4.4582	3
3	5.0509	2
4	6.117	2

Monte Carlo sampling of impulse capability

A 50-sample Monte Carlo experiment varies the pursuer maximum impulse uniformly over 50-70 m/s and the evader maximum impulse over 20-30 m/s. The reported runs all converge; the observed tendency is shorter capture time where the pursuer has a larger manoeuvre-capability advantage. These results describe numerical performance over the stated sampling ranges, rather than proving convergence for arbitrary initial conditions.

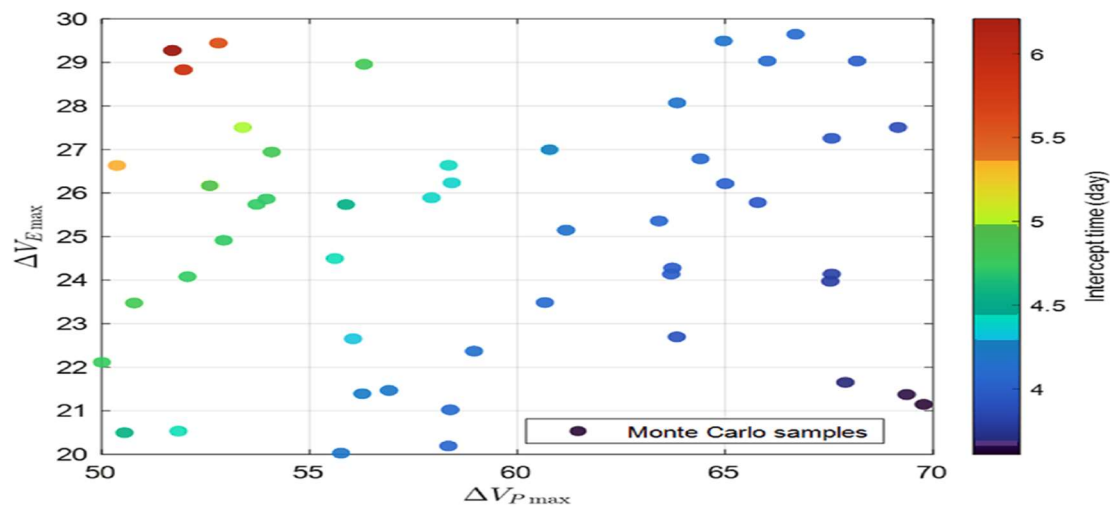


Figure 13: Interception time as a function of pursuer and evader impulse capabilities

Numerical sensitivity of the initialization

The grid-resolution and population-size experiments preserve the input configurations and impulse vectors reported in Table 5. Increasing coarse-grid divisions from 80 to 90 increases reported CPU time from 22 to 34 minutes. Increasing the population from 24 to 26 raises it

from 22 to 37 minutes. With a population of 10, the computation is faster but some candidate impulse components differ markedly from those obtained using 24 individuals. Accordingly, feasible convergence should be distinguished from insensitivity of the inferred optimum.

Table 5. Sensitivity of Grid Resolution and Population To Initial Guesses

Grid Quantity	Population	CPU Time (min)	Pursuer Δv (m/s)	Evader Δv (m/s)
80	24	22	(64.39; -27.39; -1.89)	(36.60; -16.11; -1.08)
			(58.03; -39.13; 1.11)	(31.62; -24.49; 0.23)
			(-8.87; -69.39; 2.48)	(-4.39; -39.68; 2.39)
			(-46.46; -52.36; 0.31)	(-26.53; -29.94; 0.18)
90	24	34	(67.54; -15.11; 10.47)	(39.84; -3.39; 0.98)
			(55.29; -40.88; -13.11)	(30.21; -25.46; -6.21)
			(-7.42; -69.31; -6.37)	(-4.53; -39.58; -3.57)
			(-45.58; -52.87; -5.18)	(-26.17; -30.08; -3.23)
80	10	14	(68.81; -12.45; 3.05)	(39.98; -0.68; -0.92)
			(53.27; -44.97; -6.27)	(28.74; -27.58; -3.60)
			(-5.13; -69.32; -8.26)	(-0.77; -39.35; -7.16)
			(-46.11; -52.63; -1.96)	(-26.29; -30.12; -1.07)
80	26	37	(64.39; 27.39; -1.89)	(39.96; -0.67; 1.61)
			(55.60; -37.36; -20.31)	(31.16; -23.54; -8.64)
			(-1.89; -69.13; -10.84)	(-1.13; -39.58; -5.70)
			(-43.51; -54.22; -8.16)	(-24.88; -30.97; -4.62)

Computational comparison with grid enumeration

For the co-orbital test case, a fixed-angle grid search uses 30° steps in direction and 100/T time steps. The study reported 51.9305 minutes for the exhaustive search and 5.3737 minutes for the hybrid approach, corresponding to

an approximate order-of-magnitude reduction in execution time. This runtime comparison applies only to the reported grid resolution, implementation and co-orbital benchmark; hardware specifications and repeat-run variation were not reported.

Notation and Symbols

Symbol	Definition
P, E	pursuer and evader spacecraft identifiers
i, j	spacecraft index and manoeuvre-round index
t_s, t_c	initial epoch and capture epoch
$S\{i, j\}$	epoch of manoeuvre j by spacecraft i
$\Delta obs, \Delta t_m, n_m$	observation delay, manoeuvre interval and number of rounds
Q_i, u_i	position and velocity vectors
$\Delta u\{i, j\}, \bar{u}_{imp, i}$	impulsive velocity increment and maximum magnitude
$\varphi, \varphi_0, n_{orb}, P_{orb}$	phase, reference phase, angular rate and orbital period
X, Y, Z	rotating-frame Cartesian components
v, GM_E, GM_M	mass fraction and the primaries' gravitational parameters
D_{EM}, L^*, T^*	Earth-Moon separation, characteristic length and time
ρ_1, ρ_2	scalar distances to the primary bodies
A_i	rotating-frame acceleration vector
J, H_i	performance functional and Hamiltonian
P_q, i, p_u, i	position and velocity costates
Ψ_f, ξ	terminal functional and terminal multiplier
$\gamma_{imp, i}$	impulse-bound multiplier for spacecraft i
$S_{qu}(t), e_q, \varepsilon_q$	state sensitivity, terminal position residual and tolerance
Δu_{0corr}	linearized initial-velocity correction
A_P, A_E, t_c	feasible strategy sets and equilibrium capture time

Discussion

The numerical results show that the circular restricted three-body problem provides a useful framework for analysing impulsive pursuit and evasion when the gravitational influence of the Moon substantially alters spacecraft motion. Unlike a local two-body approximation, the rotating frame model admits trajectories whose geometry and interception opportunities change considerably with orbital phase. The dynamical structures associated with Earth and Moon libration point motion provide a physical context for such trajectories (Ross et al., 2004). Orbital pathways therefore matter as much as initial separation.

The departure-phase analysis illustrates this dependence most clearly. With the evader held on its reference orbit and the pursuer limited to a 50 m/s impulse, the computed interception paths fall into outward transfer, orbital loitering and lunar gravity-assisted classes. Their occurrence at different departure-phases suggests that phase affects access to dynamically favourable pathways and the time required to exploit them. This interpretation accords with earlier treatments of nonlinear cislunar transport, although the present simulations do not isolate a particular invariant manifold or establish its role in each computed path (Campana et al., 2024). Unresolved phases do not establish physical impossibility outside the tested limits.

The single-impulse games further show why the initial orbital configuration cannot be separated from manoeuvring authority. The reported capture time is 4.5529 days in the co-orbital case and 15.5334 days when

the spacecraft occupy distinct distant retrograde orbits. The latter encounter includes lunar gravity-assisted arcs and a nonmonotonic separation history. These results are consistent with the broader observation that orbital pursuit and evasion is sensitive to nonlinear encounter geometry. Nevertheless, the two examples use different initial states and different impulse limits. The longer capture time in the distinct orbit case therefore cannot be attributed solely to orbital separation or to a difference in available thrust. Their contributions require controlled comparison.

The repeated manoeuvre experiments clarify the additional role of decision frequency. Three manoeuvre rounds in the co-orbital case yield a capture time of 4.4582 days, while four rounds in the distinct orbit case give 6.5457 days. The impulse interval experiment provides the clearer comparison: increasing the interval from one to four days changes capture time from 3.6314 to 6.117 days, while the number of rounds decreases from four to two. More frequent opportunities permit the pursuer to correct its path earlier, but they also enlarge the decision and computation burden. Manoeuvre timing is also central in previous impulsive game formulations.

Information structure is particularly important in interpreting the game solution. The evader acts first, and the pursuer responds after observing the evader manoeuvre, so the reported sequential strategy is formulated as a Stackelberg game. The simultaneous decision comparison instead concerns a static Nash setting with a fixed terminal time and a terminal distance criterion. Consequently, any difference between those

outcomes cannot be assigned solely to the equilibrium concept because the performance criteria also differ. Classical differential game theory provides the broader mathematical basis for distinguishing admissible strategies and competing objectives. The formulation thus concerns a specific sequential minimum time game.

The computational evidence supports the practicality of the hybrid initialization procedure under the cases investigated, although it is more limited than a general robustness claim. All 50 Monte Carlo samples converged for pursuer impulse bounds between 50 and 70 m/s and evader bounds between 20 and 30 m/s. Within that range, a larger pursuer capability advantage was associated with shorter capture times. Untested conditions remain outside this evidence. Earlier numerical work on orbital pursuit and evasion has also identified the solution of nonlinear boundary value systems as a demanding component of indirect methods. The sensitivity study reinforces that concern: increasing the coarse-grid count from 80 to 90 raised runtime from 22 to 34 minutes, whereas reducing the population to 10 changed several candidate impulse components despite obtaining feasible solutions.

The comparison with conventional grid enumeration is more decisive on computational cost for the specified benchmark. The hybrid procedure took 5.3737 minutes compared with 51.9305 minutes for the fixed step grid search, an approximately tenfold reduction. Selective directional search and local refinement reduce enumeration. Comparable hierarchical approaches have been considered in spacecraft trajectory optimization. However, execution times are contingent on the computer, stopping tolerances, grid resolution and implementation. Moreover, feasibility and convergence of the shooting calculation establish satisfaction of a numerical set of conditions, not uniqueness or a globally optimal Stackelberg solution.

Several restrictions qualify the operational interpretation. The circular restricted three-body model assumes idealized primary motion, while state estimation errors, manoeuvre execution errors and uncertainty in an opposing spacecraft's intentions are absent from the simulations. Observation delay and Halo tests are not included in the Results and are not assessed here. The present findings are best understood as a computational baseline for evaluating cislunar interception strategies under known initial states, prescribed manoeuvre limits and sequential decisions, rather than a validated operational guidance law.

CONCLUSION

This study formulated a sequential impulsive pursuit-evasion game in cislunar space by integrating Earth-Moon circular restricted three-body dynamics, Pontryagin's maximum principle, and the associated multipoint

boundary-value conditions with hybrid numerical initialization and MATLAB-based graphical analysis. The reported candidate interception solutions depended on departure phase, orbit geometry, and manoeuvre constraints: the single-impulse co-orbital and distinct-orbit cases yielded capture times of 4.5529 and 15.5334 days under different impulse limits, while capture time increased from 3.6314 to 6.117 days as the manoeuvre interval increased from one to four days in the specified interval experiment. For the reported co-orbital benchmark, the hybrid computation required 5.3737 minutes versus 51.9305 minutes for grid enumeration, and all 50 sampled manoeuvre-capability cases converged within the tested ranges. These findings illustrate the usefulness of the proposed integrated equation formulation and its numerical implementation, but do not establish global Stackelberg optimality, general robustness, or operational readiness. Future work should address mathematical verification, reproducibility, higher-fidelity dynamics, and uncertainty in spacecraft states and manoeuvres.

RECOMMENDATIONS

Future research should extend the present study by incorporating higher fidelity Earth-Moon ephemeris models and additional gravitational perturbations to assess their effects on interception time, terminal accuracy and manoeuvre requirements. Further investigations should consider observation delays, state estimation uncertainties and adaptive manoeuvre scheduling to improve the practical applicability of pursuit and evasion strategies. Additionally, the proposed numerical method should be validated across different orbital configurations, impulse constraints and optimization settings. Comparisons with alternative numerical approaches, together with systematic convergence and computational efficiency assessments, would strengthen the reliability and reproducibility of the proposed framework.

REFERENCES

- Almeida, A. K. de, Oliveira, V. M. de, Vaillant, T., Maia, D., Correia, A. C. M., Barbosa, D. A., & Santos, L. C. C. dos. (2025). Connecting Earth and Moon via the L1 Lagrangian point. <https://doi.org/10.48550/arxiv.2502.11694>
- Atallah, M., & Servadio, S. (2024). Advances in Cislunar Periodic Solutions via Taylor Polynomial Maps. <https://doi.org/10.48550/arxiv.2409.03692>
- Bardhan, R., & Ghose, D. (2015). Nonlinear Differential Games-Based Impact-Angle-Constrained Guidance Law. *Journal of Guidance Control and Dynamics*, 38(3), 384–402. <https://doi.org/10.2514/1.g000940>

- Bowerfind, S. R., & Taheri, E. (2024). Rapid Approximation of Low-Thrust Spacecraft Reachable Sets within Complex Two-Body and Cislunar Dynamics. *Aerospace*, 11(5), 380. <https://doi.org/10.3390/aerospace11050380>
- Campana, C., Merisio, G., & Topputo, F. (2024). Low-energy Earth–Moon transfers via Theory of Functional Connections and homotopy. *Celestial Mechanics and Dynamical Astronomy*, 136(3). <https://doi.org/10.1007/s10569-024-10192-5>
- Elizabeth, G., Mikayla. (2025). Global Characterization of Optimal Trajectory Behavior in the Cislunar Regime Via Data-Driven Learning [Figshare (United Kingdom)]. In Figshare. <https://doi.org/10.25394/pgs.30828647.v1>
- Ferrara, F., Arana, L. W. S., Dörfler, F., & Li, S. H. Q. (2025). A Markov Decision Process Framework for Early Maneuver Decisions in Satellite Collision Avoidance. <https://doi.org/10.48550/arxiv.2508.05876>
- Fotiadis, F., Rommel, Q., Falco, G., & Topcu, U. (2025). Adversarial Pursuits in Cislunar Space. <https://doi.org/10.48550/arxiv.2509.20330>
- Fu, S., Gong, S., & Shi, P. (2024). Analytical Pursuit-Evasion Game Strategy in Arbitrary Keplerian Reference Orbits. <https://doi.org/10.48550/arxiv.2411.15912>
- Fu, S., Gong, S., & Shi, P. (2025). Analytical pursuit-evasion game strategy in arbitrary Keplerian reference orbits. *Aerospace Science and Technology*, 158, 109946. <https://doi.org/10.1016/j.ast.2025.109946>
- Gao, A., Deng, K., Wang, J., & Zhao, Z. (2025). Hierarchical Intelligent Optimization Decision Method for Multi-Spacecraft Pursuit-Evasion Orbital Game. *IFAC-PapersOnLine*, 59(20), 1782–1787. <https://doi.org/10.1016/j.ifacol.2025.11.417>
- Gupta, K., Allen, R., Fridovich-Keil, D., & Topcu, U. (2025). More Information Is Not Always Better: Connections Between Zero-Sum Local Nash Equilibria in Feedback and Open-Loop Information Patterns. *IEEE Control Systems Letters*, 9, 1405–1410. <https://doi.org/10.1109/lcsys.2025.3583329>
- Hafer, W. T., Reed, H. L., Turner, J. D., & Pham, K. (2015). Sensitivity Methods Applied to Orbital Pursuit Evasion. *Journal of Guidance Control and Dynamics*, 38(6), 1118–1126. <https://doi.org/10.2514/1.g000832>
- Halsey, S., Curtis, D., & Prince, E. R. (2024). Impact of Disparate Goals on Evader Performance in an Orbital Pursuer-Evader Game. <https://doi.org/10.2514/6.2024-2070>
- Han, H., & Dang, Z. (2023). Models and Strategies for J 2 - Perturbed Orbital Pursuit–Evasion Games. *Space Science & Technology*, 3. <https://doi.org/10.34133/space.0063>
- He, H., Shi, P., & Zhao, Y. (2022). Hierarchical Optimization Algorithm and Applications of Spacecraft Trajectory Optimization. *Aerospace*, 9(2), 81. <https://doi.org/10.3390/aerospace9020081>
- Jagat, A., & Sinclair, A. J. (2017). Nonlinear Control for Spacecraft Pursuit-Evasion Game Using the State-Dependent Riccati Equation Method. *IEEE Transactions on Aerospace and Electronic Systems*, 53(6), 3032–3042. <https://doi.org/10.1109/taes.2017.2725498>
- Kumar, B., Rawat, A. K. S., Rosengren, A. J., & Ross, S. D. (2025). Cislunar Resonant Transport and Heteroclinic Pathways: From 3:1 to 2:1 to L1. <https://doi.org/10.48550/arxiv.2509.12675>
- LaFarge, N. B., Miller, D., Howell, K. C., & Linares, R. (2020). Guidance for Closed-Loop Transfers using Reinforcement Learning with Application to Libration Point Orbits. *AIAA Scitech 2020 Forum*. <https://doi.org/10.2514/6.2020-0458>
- Li, R., Xue, D., Lang, X., Liu, X., & Chen, Z. (2025). Differential Game Approach to Orbital Pursuit-Evasion Problems. *IFAC-PapersOnLine*, 59(20), 1948–1952. <https://doi.org/10.1016/j.ifacol.2025.11.443>
- Liu, Z., Xu, S., Zhao, L., & Dang, Z. (2025). Intermittent Pursuit of Noncooperative Space Targets under Discontinuous Observations via Deep Reinforcement Learning. *IEEE Transactions on Aerospace and Electronic Systems*, 62, 2109–2129. <https://doi.org/10.1109/taes.2025.3636848>
- Mehlman, C., Kounios, A. A., Lai, L., Prasad, A. R., Kully, W., Brown, D., Hughes, R., Chalamalasetty, S., Distler, J., Dilone, R., Palomino, E., Lewick, K., Calabrese, M., Boschetti, N., Goel, M., Phillips, S., & Falco, G. (2025, January 1). The MAGPIE: Satellite Autonomy for Uncooperative Environments. *Proceedings of the ... Annual Hawaii International Conference on System Sciences/Proceedings of the Annual Hawaii International Conference on System Sciences*. <https://doi.org/10.24251/hicss.2025.870>
- Paladini, S. (2026). The Need for Cislunar Governance: Environmental Security and Sustainability Challenges. *International Journal for the Semiotics of Law - Revue*

Internationale de Sémiotique Juridique.
<https://doi.org/10.1007/s11196-026-10510-0>

Pontani, M., & Conway, B. A. (2009). Numerical Solution of the Three-Dimensional Orbital Pursuit-Evasion Game. *Journal of Guidance Control and Dynamics*, 32(2), 474–487. <https://doi.org/10.2514/1.37962>

Ross, S. D., Koon, W. S., Lo, M. W., & Marsden, J. E. (2004). Application of dynamical systems theory to a very low energy transfer. In CaltechAUTHORS (California Institute of Technology). California Institute of Technology. <https://authors.library.caltech.edu/20297/>

Starek, J. A., Schmerling, E., Maher, G. D., Barbee, B. W., & Pavone, M. (2016). Fast, Safe, Propellant-Efficient Spacecraft Motion Planning Under Clohessy–Wiltshire–Hill Dynamics. *Journal of Guidance, Control, and Dynamics*, 40(2), 418–438. <https://doi.org/10.2514/1.g001913>

Venigalla, C., & Scheeres, D. J. (2021). Delta-V-Based Analysis of Spacecraft Pursuit–Evasion Games. *Journal of Guidance Control and Dynamics*, 44(11), 1961–1971. <https://doi.org/10.2514/1.g005901>

Wei, Y., Liu, T., Wei, C., Zhang, R., & Gu, H. (2023). Two Hierarchical Guidance Laws of Pursuer in Orbital Pursuit–Evasion–Defense Game. *Aerospace*, 10(8), 668. <https://doi.org/10.3390/aerospace10080668>

Weintraub, I. E., Pachter, M., & García, E. (2020). An Introduction to Pursuit-evasion Differential Games. <https://doi.org/10.23919/acc45564.2020.9147205>

Xu, S., Zhao, L., Zhang, W., & Dang, Z. (2025). Delta-V-Based Cooperative Strategies for Orbital Two-Pursuer

One-Evader Pursuit–Evasion Games. *Space Science & Technology*, 5. <https://doi.org/10.34133/space.0222>

Yan, M., Yang, R., Zhao, Y., Yue, L., & Zhao, X. (2024). Exoatmospheric Evasion Guidance Law with Total Energy Limit via Constrained Reinforcement Learning. *International Journal of Aeronautical and Space Sciences*, 25(4), 1361–1379. <https://doi.org/10.1007/s42405-024-00722-8>

Yan, R., Shi, Z., & Zhong, Y. (2019). Task Assignment for Multiplayer Reach–Avoid Games in Convex Domains via Analytical Barriers. *arXiv (Cornell University)*, 36(1), 107–124. <https://doi.org/10.1109/tro.2019.2935345>

Yang, X., & Gao, J. (2013). Uncertain differential games with application to capitalism. *Journal of Uncertainty Analysis and Applications*, 1(1). <https://doi.org/10.1186/2195-5468-1-17>

Ye, D., Shi, M., & Sun, Z. (2020). Satellite proximate pursuit-evasion game with different thrust configurations. *Aerospace Science and Technology*, 99, 105715. <https://doi.org/10.1016/j.ast.2020.105715>

Zeeuw, A. de. (2024). A Crash Course in Differential Games and Applications. *Environmental and Resource Economics*, 87(6), 1521–1544. <https://doi.org/10.1007/s10640-024-00844-3>

Zhang, L., Ghimire, M., Zhang, W., Xu, Z., & Ren, Y. (2024). Value Approximation for Two-Player General-Sum Differential Games With State Constraints. *IEEE Transactions on Robotics*, 40, 4631–4649. <https://doi.org/10.1109/tro.2024.3411850>