



A Computational and Statistical Analysis of the Relationship Between Euler's Totient Function and the Sum-of-Divisors Function

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KEYWORDS

Euler's Totient Function,
Sum-of-Divisors Function,
Number Theory,
Statistical Analysis,
Pearson Correlation,
Linear Regression,
MATLAB,
Arithmetic Functions,
Prime Factorization.

ABSTRACT

This study examines the statistical relationship between Euler's Totient Function, $\phi(n)$, and the Sum-of-Divisors Function, $\sigma(n)$, using computational and statistical methods implemented in MATLAB. The analysis was conducted on the first 1,000 positive integers, for which both arithmetic functions were computed. Descriptive statistics, covariance, Pearson's correlation coefficient, statistical significance testing, linear regression, and graphical analysis were employed to investigate the relationship between the two functions. The results showed that the mean value of $\phi(n)$ was 304.1920 whereas the mean value of $\sigma(n)$ was 823.0810. The Pearson's correlation coefficient was calculated and there was a positive but relatively weak-to-moderate linear association, $r=0.3443$. There was a significant relation between the two ($p < 0.001$). A simple linear regression was used to get the model as $\sigma(n)=0.8737+\phi(n)=557.3142$ and the coefficient of determination $R^2 = 0.1185$. This means that there is about 11.85% of the variance in the Sum-of-Divisors Function accounted for by the simple linear relationship with Euler's Totient Function. The results demonstrate that there is a positive and statistically significant relationship between $\phi(n)$ and $\sigma(n)$; however, the relationship is not strongly linear. This may be attributed to the distinct arithmetic properties measured by the two functions and their different dependencies on the prime factorization and divisor structure of integers. Overall, the study highlights the value of computational and statistical methods implemented in MATLAB for exploring relationships between classical arithmetic functions.

CITATION

Zwain, H. M. (2026). A Computational and Statistical Analysis of the Relationship Between Euler's Totient Function and the Sum-of-Divisors Function. *Journal of Science Research and Reviews*, 3(5), 312-321. <https://doi.org/10.70882/josrar.2026.v3i5.309>

INTRODUCTION

Number theory is a branch of mathematics that studies the properties and relationships of integers (Cox et al., 2022). It encompasses several important topics, including the study of arithmetic functions, which assign numerical values to positive integers according to specific properties of those integers (Baker, 2022). Euler's Totient Function, $\phi(n)$, and the Sum-of-Divisors Function, $\sigma(n)$, are two classical arithmetic functions of particular interest.

Euler's Totient Function, $\phi(n)$, is defined as the number of positive integers less than or equal to (n) that are relatively prime to (n) . It plays an important role in various areas of number theory and has applications in several fields of mathematics, including modular arithmetic and cryptography (Kaliski, 2025). The Sum-of-Divisors Function, $\sigma(n)$, on the other hand, is defined as the sum of all positive divisors of (n) . This function provides

information about the divisor structure of an integer and is closely related to its prime factorization (Jakimczuk, 2026). Both functions characterize different aspects of the arithmetic structure of integers and depend on properties of their prime factors; however, they measure fundamentally different quantities. This raises an interesting mathematical question: Is there a statistical relationship between the values of $\phi(n)$ and $\sigma(n)$? (Cox et al., 2022).

With the development of computational software, large amounts of numerical data can be generated and analyzed efficiently. In this study, MATLAB is used to compute $\phi(n)$ and $\sigma(n)$ for the first 1,000 positive integers and to investigate the statistical and graphical relationship between the two functions. The relationship is quantified using descriptive and inferential statistical measures, including the mean, variance, standard deviation, covariance, Pearson's correlation coefficient, (p)-value, and coefficient of determination. In addition, simple linear regression and graphical analysis are employed to determine whether a linear model provides an appropriate representation of the relationship between the two arithmetic functions.

The aim of this study was to computationally and statistically characterize the relationship between Euler's totient function $\phi(n)$ and the sum-of-divisors function $\sigma(n)$ over the first 1,000 positive integers using descriptive statistics, correlation, linear regression and graphical analysis.

Problem Statement

The values of mathematical functions Euler's function $\phi(n)$ and the sum of divisors function $\sigma(n)$ depend on the structure of the numbers and are vital. Since both functions are based on the prime factorization of n , it makes sense to explore if there is any measurable statistical relationship between the values of the two functions.

The two functions, however, have different properties of numbers. It is not clear whether there is a close relationship between the two or whether they are just highly correlated due to the fact that both depend on prime factors. A close relationship between these two functions or only a limited correlation because they are both dependent on prime factors is not clear. So the primary concern of this study is to find the statistical strength, direction and significance between $\phi(n)$ and $\sigma(n)$ for a significant number of positive integers.

The primary goal of this study is to identify the magnitude and direction of association (as well as the strength) between $\phi(n)$ and $\sigma(n)$ on a large sample of positive integers. In particular, this study investigates if there is a suitable linear model that can be used for this relationship between these two functions. It is also an inquiry to whether the variance in $\sigma(n)$ can be accounted for by $\phi(n)$

and if it would clearly be visible in the statistical analysis and the graphical illustration of differences between $\phi(n)$ and $\sigma(n)$ that there would be any patterns discernible between the two functions. Hence, this problem was mathematically analyzed based on MATLAB software, analysis of the obtained data was performed by descriptive statistics, correlation analysis, hypothesis testing, linear regression and graphical method with first 1000 positive integers, and then the mathematical results were evaluated.

Research Objectives

This research primarily aims to study the statistical relationship between Euler's function $\phi(n)$ and its sum of denominators $\sigma(n)$ using computational and statistical methods.

The sub-objectives include:

- i. Calculating Euler's function $\phi(n)$ and its sum of denominators $\sigma(n)$ for the first 1000 positive integers using MATLAB.
- ii. Describing the numerical behavior of the two functions using descriptive statistical measures, including the mean, variance, and standard deviation.
- iii. Determining the covariance between $\phi(n)$ and $\sigma(n)$ and identifying the direction of this covariance.
- iv. Measuring the strength of the linear relationship between the two functions using Pearson's correlation coefficient.
- v. To test the statistical significance of the observed correlation using the corresponding p-value.
- vi. To develop a simple linear regression model to investigate the feasibility of using $\phi(n)$ to predict changes in $\sigma(n)$.
- vii. Evaluate the explanatory power of the regression model using the coefficient of determination (R^2).
- viii. Represent the relationship and behavior of the two functions using scatter plots, regression lines, function plots, histograms, and comparison graphs.
- ix. Interpret the statistical results from a number theory perspective, determining how prime factorization and the divisor structure of integers contribute to the observed relationship.
- x. Evaluate whether a simple linear relationship adequately describes the relationship between the Euler function and the sum of divisors function.

Related Studies

In recent years, various research work has been carried on Sum of Divisors Function ($\sigma(n)$), Euler's Totient Function ($\phi(n)$) and relationship between sum of divisors function and Euler's totient function in number theory have been seen. The present research is based on the above studies, it is important to note that these studies provide

the theoretical and computational basis of the present research.

The most direct study was Computational and Statistical Study of the Relationship Between $(\sigma(n))$ and $(\phi(n))$ (Zwain, 2026) where the relationships of these arithmetic functions was studied numerically and statistically with MATLAB over large ranges of natural numbers. It is especially relevant, as they show the usefulness of computational and statistical methods. The present study, however, is only meant to determine the direct linear relationship of $(\sigma(n))$ to $(\phi(n))$ with the help of correlation and regression analysis.

Oliver et al., (2023) used to study the asymptotic identities involving sums of the divisors and its theoretical behavior and the relationship to other mathematical problems. Amdeberhan, et al (2024) presented a general extension of the classical divisor-sum theory, by means of generating functions, divisibility properties, and combinatorial identities. The function has a complicated structure when

viewed as an arithmetic function, as shown by Johnson (2026) who studied repeated values of $(\sigma(n))$ and their natural density. Using computational methods, Cox, Ghosh and Sultanow investigated generalized divisor sums and gave classifications as deficient, perfect and abundant integers and correlated $(\sigma(n))$ with $(\phi(n))$. Taken together, these studies demonstrate the significance of $(\sigma(n))$ and $(\phi(n))$ in the multiplicative number theory, divisor structures, and computational mathematics. Most of the studies have, however, focused on theoretical properties and/or asymptotic behavior, classifications or "generalized functions" rather than direct statistical measurement of their relationship.

For filling this gap, the present study utilizes MATLAB to analyze the first 1000 positive integers and employs descriptive statistics, covariance, Pearson correlation, hypothesis testing, linear regression and graphics to statistically analyses the relationship between $(\phi(n))$ and $(\sigma(n))$.

Table 1: Comparison Table

Ref.	Main Focus	Methodology	Role of $\phi(n)$	Role of $\sigma(n)$	Main Contribution	Relation to Present Study
Zwain, (2026)	Relationship between $\sigma(n)$ and $\phi(n)$	Computational and statistical analysis using MATLAB	Directly investigated	Directly investigated	Examined the behavior of $\sigma(n)/\phi(n)$ across numerical ranges	Most closely related study; present work focuses on direct correlation and linear regression
Oliver et al., (2023)	Asymptotic identities for divisor sums	Theoretical and asymptotic analysis	Not the primary focus	Central to the study	Developed asymptotic results and investigated additive convolutions of divisor sums	Provides theoretical background on the behavior of divisor sums
Amdeberhan et al., (2024)	Extensions of MacMahon's sums of divisors	Generating functions, divisibility theorems, and combinatorial analysis	Not the primary focus	Central to the study	Developed extensions and new combinatorial identities related to divisor sums	Provides theoretical foundation for the mathematical structure of $\sigma(n)$
Johnson, (2026)	Equal values and natural density of $\sigma(n)$	Multiplicative number theory and density analysis	Used as part of the theoretical framework	Central to the study	Investigated repeated values of $\sigma(n)$ and their arithmetic structure	Demonstrates the complex behavior of $\sigma(n)$ but does not directly measure correlation
Cox et al., (2022)	Generalization and computational applications of $\sigma(n)$	Computational analysis using MATLAB	Used in part of the proposed methods	Central to the study	Studied deficient, perfect, and abundant numbers and generalized divisor functions	Supports the computational approach but has a different research objective
Present Study	Statistical relationship between $\phi(n)$ and $\sigma(n)$	MATLAB descriptive statistics correlation regression visualization	Independent variable	Dependent variable	Quantifies correlation, statistical significance, and linear explanatory power	Extends previous work through a direct statistical investigation

Research Gap

The results of the literature survey reveal that theory and computation of Sum-of-Divisors Function has engaged a great deal of interest among the researchers and practitioners. In fact, some previous studies have investigated the divisor sums under various aspects, namely under the asymptotic identity, combinatorial structure, repeated value, natural density, number classification, and generalized arithmetic function viewpoints (Oliver et al., 2023; Amdeberhan et al., 2024; Johnson, 2026; Cox et al., 2022). The most similar work in this regard (Zwain, 2026) studied the computational and statistical characteristics of the relationship between $\sigma(n)$ and $\phi(n)$, especially the ratio $\sigma(n)/\phi(n)$.

The present work has made contributions; but there is a need for a more direct statistical analysis of the relation between the two functions. Specifically, one would like to measure the strength and direction of their linear association, make statistical inferences about the association, and know how well variations in $\sigma(n)$ can be accounted for in terms of variations in $\phi(n)$ by using a linear regression model.

Based on this, the present study focuses on carrying out the first 1000 positive integer numbers by systematically analyzing them statistically by using MATLAB. Otherwise study involves calculation of the two Arithmetic functions along with Descriptive statistics, covariance, Pearson's correlation coefficient, p-value analysis, linear regression, R2 and graphical visualization. Here, the quantitative understanding of the relationship between $\phi(n)$ and $\sigma(n)$ is discussed, which complements theory and computation, as detailed in previous papers.

MATERIALS AND METHODS

Research Design

This study employs a computational and quantitative research design to investigate the statistical relationship between Euler's function, $\phi(n)$, and the sum of divisors function, $\sigma(n)$. The entire analysis was performed using MATLAB software, which was used for data generation, calculating the functions, statistical analysis, regression modeling, and data visualization.

The study considers the first 1000 positive integers, $n = 1, 2, \dots, 1000$. The values of $\phi(n)$ and $\sigma(n)$ for each integer in this range was computed and entered as datasets of numbers. These datasets were then analyzed in order to detect patterns and relate the quantitative comparison of the two functions.

All computational procedures in this study were implemented using MATLAB R2018a

Computational Data Generation

The first step in the computational procedure is that the sequence of positive integers ranging from 1 to 1,000 is

created. For every integer n , there are two numbers to be computed:

- i. Euler's Totient Function, $\phi(n)$
- ii. Sum-of-Divisors Function, $\sigma(n)$

The Euler's Totient Function is the number of positive integers less than or equal to n that are relatively prime to n . But when $n > 1$ it can be computed by the distinct prime factors of n : $\phi(n) = n \prod_{p|n} (1 - 1/p)$.

The special case $\phi(1) = 1$ is explicitly considered.

The Sum-of-Divisors Function is defined by; $\sigma(n) = \sum_{d|n} d$,

Thus, for each positive integer n , the value of $\sigma(n)$ is equal to the sum of its positive divisors.

The computed matrices are stored in two matrices, and easily manipulated with the entire set of numbers and statistics in MATLAB.

Descriptive Statistical Analysis

The first part of descriptive statistics is to summarize the behavior of the two functions. For both $\phi(n)$ and $\sigma(n)$ the following measures are computed:

- i. Medians, to find numbers that also determine the central tendency of the data.
- ii. Variance: To establish the total spread from the mean.
- iii. Standard deviation: a measure of the deviation of the values from the mean value.

These will help to compare the numerical nature and differences between the two functions in a designated interval.

In addition, the covariance of the two sets of data is computed. To find out if the two functions increase or decrease together, or if one increases while the other decreases, you need to use covariance.

Correlation Analysis

The correlation of Pearson is a main measurement of linear association amongst $\phi(n)$ and $\sigma(n)$. It is computed based on

$$r = \text{Cov}(\phi, \sigma) / s_{\phi} s_{\sigma}$$

where s_{ϕ} and s_{σ} represent the standard deviations of the two variables.

r can be between the values of -1 and 1. If the value is positive, then the linear association is positive; if the value is negative, then the linear association is negative. The closer the number is to zero, the less it will tend to be linearly related to the other numbers.

A hypothesis test will be used with the appropriate p-value to see if the correlation that was found is statistically significant or not. The null hypothesis is defined as:

$H_0: \rho = 0$, where ρ represents the population correlation coefficient.

$H_1: \rho \neq 0$.

The level of significant value, $\alpha = 0.05$ was used. If this resulting probability number is smaller than the 0.05 value, then the null hypothesis can be rejected and provide a

statistical supporting statement for linear correlation between the two functions.

Linear Regression Model

A simple linear regression was carried out to obtain a closer relationship between the two functions. In this model, $\phi(n)$ is thought of as the explanatory variable and $\sigma(n)$ as the response variable.

Here is a regression model, modeled as

$$\sigma(n) = \beta_0 + \beta_1 \phi(n) + \epsilon,$$

Where β_0 is the constant term, β_1 is the slope, and ϵ represents the residual error.

The regression coefficients are estimated using MATLAB. The matching model provides an estimate of how changes in $\phi(n)$ relate to changes in $\sigma(n)$.

The R^2 is also determined to estimate the explanatory power of the regression model. Measures the percentage of variance in $\sigma(n)$ as a linear function of $\phi(n)$ that is predictable.

Graphical Analysis

The relationship and distribution between the two functions is determined both from a graphical analysis and from the numerical results.

To explore the general trend of association between the variables a scatter graph of $\phi(n)$ against $\sigma(n)$ is plotted. A fitted regression line is graphed on the graph to illustrate linear relationship: direction and strength.

Other plots are drawn to study the behavior for individual arithmetic functions. These are plots of $\phi(n)$ against n and of $\sigma(n)$ against n ; with these graphs, changes and patterns in $\phi(n)$ and $\sigma(n)$ for the given interval can be seen.

The histograms of both functions are drawn in the same way, and examined for distribution and dispersion of the values. Lastly, there is a graph of the behavior of both $\phi(n)$ and $\sigma(n)$ between the first 100 positive integers.

Analytical Framework

It's a combination of number-theoretic computations and statistical analysis. The two stages computations are: first, computing the 2 arithmetic functions; second computing their association, predictive relationship, i.e. the statistical evaluation.

The analysis is oriented in three complementary questions:

- i. What is the statistical behavior of $\phi(n)$ and $\sigma(n)$ in the first 1000 positive integers?
- ii. Is there a linear association between the two functions, that is, is there a linear relationship between them that is statistically significant?
- iii. How well can the variation in $\sigma(n)$ be approximated by $\phi(n)$ using a straight line model?

The data below systemically answers these questions through descriptive statistics, correlation analysis,

hypothesis testing, regression modeling, and descriptive visualization.

Reproducibility and Scope

The same set of 1,000 positive integers were used with all the computational procedures that have been implemented in MATLAB. This creates a framework for computation that can be reproduced so that the same (arithmetic) functions and statistic calculations can be repeated with the same settings.

Only the first 1,000 positive integers have been considered in this analysis, and only the linear statistical correlation between $\phi(n)$ and $\sigma(n)$ are explored. Hence, the results we conclude are those for this computational data set and we cannot state that the same linear pattern or correlation always exists for all positive integers.

In general, the present method offers a systematic and consistent way to investigate the statistical dependency between two of the classic arithmetic functions, and maintains the link between the computation itself and the associated interpretation of the number theoretical well.

RESULTS AND DISCUSSION

Descriptive Statistical Results

The first 1000 positive integers were used for the statistical analysis. Euler's Totient Function $\phi(n)$ and Sum-of-Divisors Function $\sigma(n)$ were computed for every other integer n in MATLAB. This data has then been used to make inferences and descriptive statistics.

The mean value of Euler's Totient Function is 304.1920 and that of Sum-of-Divisors Function is 823.0810. The difference is due to the different mathematical properties that can be extracted from the two functions. Multiplicative functions $\phi(n)$ and $\sigma(n)$ count the positive integers relatively prime to n and count the sum of all the positive divisors of n , respectively.

The variance of $\phi(n)$ was 50366.6077, with a standard deviation of 224.4251. In comparison, $\sigma(n)$ had a variance of 324393.1216 and a standard deviation of 569.5552. The standard deviation and variance of $\sigma(n)$, which is quite large, suggests that the Sum-of-Divisors Function is quite variable over the specified range.

The difference can be accounted for by the structure of the two functions in the arithmetic sense. Numbers with many factors may yield, on average, large numbers of $\sigma(n)$, which can cause significant variations.

Correlation Analysis

To examine the relationship between the two functions, Pearson's correlation coefficient was calculated, yielding a value of $r = 0.3443$.

A positive value for r indicates a positive linear correlation between $\phi(n)$ and $\sigma(n)$. Generally, larger values of $\phi(n)$ tend to correlate with larger values of $\sigma(n)$. However, the

magnitude of the correlation suggests that this correlation is not strong.

A correlation coefficient of approximately 0.34 indicates a weak to moderate linear relationship. Thus, although the two functions show a tendency to increase together, there is considerable variance in their individual values. Since it corresponds to the two functions representing different characteristics of numerical theory.

The positive covariance; given as 44004.3488, further supports the positive trend in the relationship. Covariance though is not a uniform measure of the strength of this relation, so the Pearson's correlation coefficient is a better measure of the correlation.

Statistical Significance

The p-value produced in Matlab was 0.000000, which is a very small p value thus less than 0.001.

Using a significance level of 0.05, the null hypothesis of no linear correlation can be rejected. This provides strong statistical evidence that the observed positive correlation between $\phi(n)$ and $\sigma(n)$ is statistically significant for the analyzed dataset.

However, statistical significance should not be confused with the strength of the relationship. The small p-value is partly due to the relatively large sample size (1000 observations). The correlation coefficient itself, $r = 0.3443$, indicates that the relationship is still relatively small.

Therefore, the results show a statistically significant correlation, but they do not indicate a strong linear relationship or a causal relationship between the two functions.

Linear Regression Analysis

A simple linear regression model was applied to study the relationship between the two functions. Euler's function was considered the explanatory variable, while the sum of divisors function was considered the respondent variable. The resulting regression equation was: $\sigma(n) = 0.8737 \phi(n) + 557.3142$

The positive slope of 0.8737 indicates that, according to the applied linear model, a one-unit increase in $\phi(n)$ is

associated with an estimated increase of approximately 0.8737 units in $\sigma(n)$.

The y-intercept value was 557.3142. This value represents the expected value of $\sigma(n)$ when $\phi(n)$ is zero within the mathematical structure of the regression model. Since $\phi(n)$ does not take a value of zero for positive integers, the y-intercept value should be interpreted primarily as a mathematical coefficient of the applied regression line, rather than as a direct theoretical interpretation of the numbers. The graph of the scatter plot seems to have a positive slope because the line has a positive correlation coefficient between the x and y values. The spread of the observations is quite large, however, around the regression line. This difference will provide evidence that $\phi(n)$ alone is not sufficient for predicting $\sigma(n)$.

Coefficient of Determination

The coefficient of determination (R^2) obtained from the regression analysis was: $R^2 = 0.1185$.

This means that approximately 11.85% of the observed variance in $\sigma(n)$ can be explained by its linear relationship with $\phi(n)$.

The relatively low value of R^2 indicates that the linear regression model has limited explanatory power. Approximately 88.15% of the variance remains unexplained by this simple linear model.

The significance of this result lies in its demonstration that statistically significant correlation should not be interpreted as evidence of a strong predictive relationship. The two functions may be statistically related, but their relationship is far more complex than can be represented by a single straight line.

Graphical Analysis

The graphical analysis conducted in MATLAB agrees with the numerical results.

The scatter graph between $\phi(n)$ and $\sigma(n)$ in figure 1 has a general positive trend. The fitted regression line is upwards (as it should be, given a positive value for the correlation coefficient). The fairly large range of the observations from the regression line shows that the linear correlation is not strong but weak to moderate.

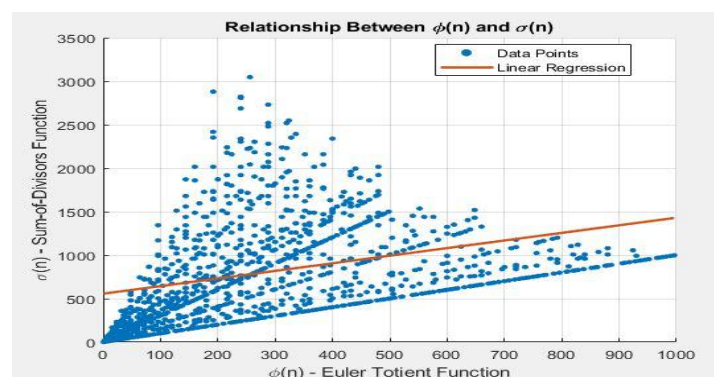


Figure 1: Relationship between $\phi(n)$ and $\sigma(n)$

As can be seen from the graph in Figure 2, Euler's Totient Function is not a monotonously increasing function of n , but something that fluctuates considerably from n to n . It is due to the variation of the prime factorization of successive integers. Between any two integers n and p that

are not necessarily equal, some may have values of $\phi(n)$ that differ by very large numbers. The value of $\phi(n)$ may differ by large numbers between two integers n and p which are not necessarily equal and which are close in magnitude.

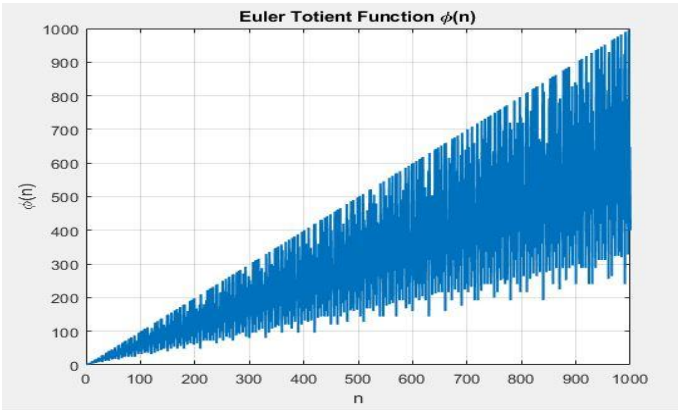


Figure 2: Euler totient function $\phi(n)$ against n

Similarly, the graph of $\sigma(n)$ versus n in Figure 3 shows considerable fluctuations. The scale of the function generally increases as the value of n increases, but this

increase is not uniform. Numbers with many divisors may have unusually large values for $\sigma(n)$, producing notable peaks in the graph.

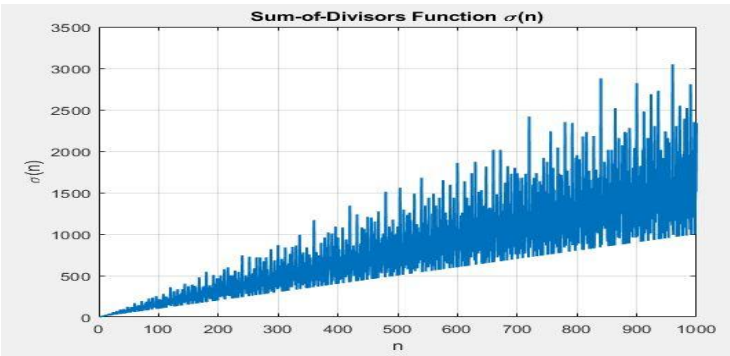


Figure 3: Sum of divisors $\sigma(n)$ against n

Figure 4 provide information about the distributions of the two functions. Euler's Totient Function $\phi(n)$ counts the number of positive integers less than or equal to (n) that

are relatively prime to (n) . It should not be confused with the prime-counting function ($\pi(n)$), which counts the number of prime numbers less than or equal to (n) .

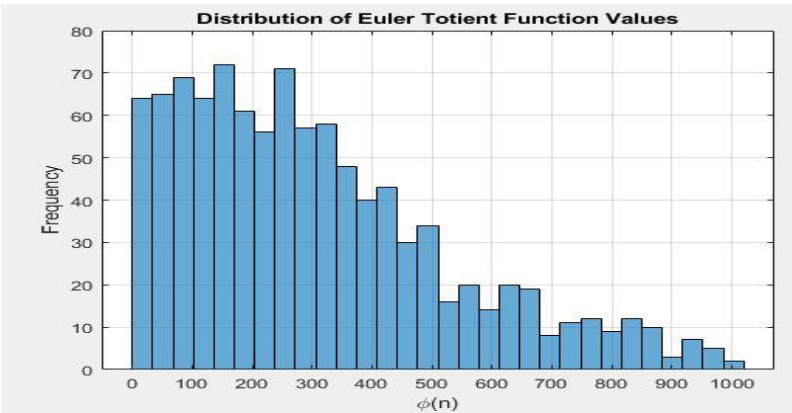


Figure 4: Distribution of euler totient function values

The variation in the sum of the denominators is illustrated in Figure 5 by the distribution of $\sigma(n)$. The variance and/or

the standard deviation of the distribution of $\sigma(n)$ is larger because of the wider spread observed.

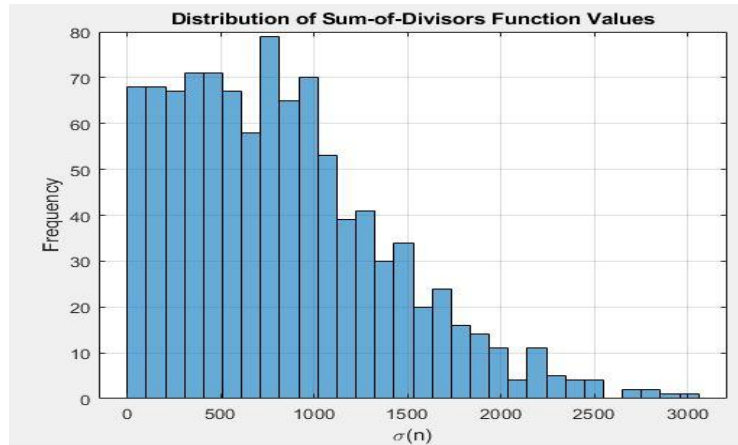


Figure 5: distribution of sum of divisors functions values

Figure 6 shows the relative graph of both functions with the first 100 integers. While the structure of the numbers is arithmetic, the values are very different, and unrelated to

that of the preceding pair for each integer. This behavior can be observed and helps to explain the positive, but weak, correlation in the overall results.

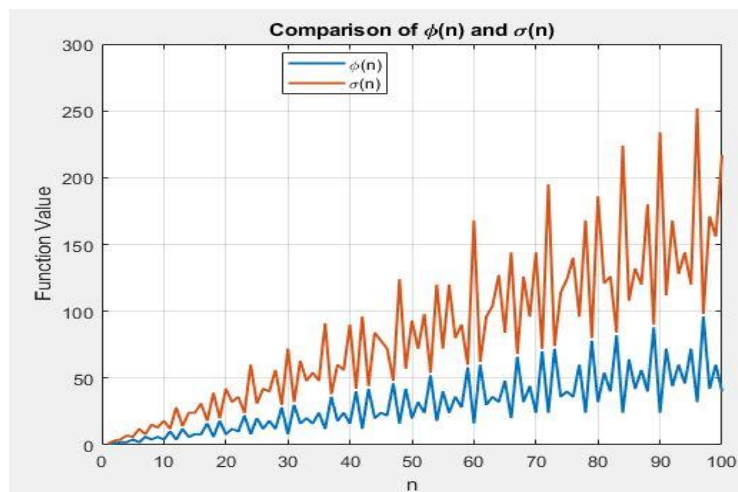


Figure 6: comparison plot for the first 100 integers

Number-Theoretic Interpretation

The observed relationship can be understood from the mathematical definitions of the two functions. Euler's Totient Function is given, for $n > 1$, by $\phi(n) = n \prod_{p|n} (1 - 1/p)$, where the product is taken over the distinct prime factors of n . On the other hand, the Sum-of-Divisors Function depends on all positive divisors of n . For an integer with prime factorization $n = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$, the function can be expressed as $\sigma(n) = \prod_{i=1}^k (1 + p_i + p_i^2 + \cdots + p_i^{a_i})$. Thus, both functions are affected by prime factorization of n , which mathematically explains the positive correlation observed in the statistical analysis. However, they utilize prime factorization in fundamentally different ways.

The coefficient function depends on different prime factors through factors of the form $1 - 1/p$, while the sum of

divisors function depends on powers of those prime factors and the structure of the perfect divisors.

Consequently, two integers with similar values of $\phi(n)$ may have significantly different values of $\sigma(n)$, and vice versa. This difference in behavior helps explain the low value of R^2 inferred from linear regression.

Overall Discussion

The MATLAB output overall shows that there is a significant infer pointwise positive relation between Euler's Totient Function and the Sum-of-Divisors Function for the first 1,000 positive integers. The correlation coefficient (r) is 0.3443, which suggests a positive, and fairly weak-to-moderate, correlation.

The regression analysis agrees with the above interpretation. The fitted regression line is positive, but $R^2 = 0.1185$ means that just 11.85% of the variations in $\sigma(n)$ is described by the simple linear relationship with $\phi(n)$.

Therefore, it is unclear from the results whether a single linear function will adequately represent the relationship between the two functions. Both are closely related by prime factorization and share a measurable statistical correlation, but they differ in their mathematical definitions and thus exhibit a considerable variance that cannot be explained using $\phi(n)$ alone.

Another aspect of the analysis is the usefulness of the combination of number-theoretic computation together with statistical methods. MATLAB is a problem-solving environment that is useful for creating large numerical data sets, evaluating arithmetical functions, performing statistical tests of data, fitting regression models, and viewing complex patterns.

Last but not least, the results are obtained from the first 1000 positive numbers and from linear correlation and regression techniques. Hence, a study with more integers, non-linear statistical models or with more number-theoretic variables may give a better understanding of the correlation between $\phi(n)$ and $\sigma(n)$.

CONCLUSION

The aim of this study is to participate in exploring the statistical relationship between Euler's Totient Function $\phi(n)$ and Sum of Divisors Function $\sigma(n)$ through the initial 1000 positive integers and computational analysis carried out using MATLAB. Arithmetic function was calculated for all integers and was subjected to descriptive statistical analysis, covariance analysis, Pearson correlation and linear regression and plotted graphically.

The results indicated that there was a positive correlation between the two functions as the Pearson correlation coefficient was obtained $r = 0.3443$. The corresponding p-value is < 0.001 and shows that the positive association seen is statistically significant. But, the moderate correlation coefficient suggests that the correlation is not strong. The linear modelling yielded the model: $\sigma(n) = 0.8737 \phi(n) + 557.3142$,

The model explains approximately 11.85% of the observed variation in $\sigma(n)$, leaving approximately 88.15% unexplained by the simple linear relationship with $\phi(n)$. Hence, the outcomes suggest that the two arithmetic features are statistically related but, certainly, the linkage is a lot more intricate than they conceptualize as a linear model.

In number theoretic terms, this association observed is due partly to both functions being related to the prime factorization of n . However, both (or both of them) are analyzing different aspects of the properties of integers, and the divisor structures of which they are referring to are

different. This is why there is a positive association, despite the general relatively weak linear association.

RECOMMENDATIONS

The present study is a first step towards a further investigation to the relation between Euler's Totient Function and the Sum-of-Divisors Function. In future studies several extensions should be considered.

In the first place, it would be possible to extend the analysis to a much wider range of integers. More than 1,000 should allow for the behavior of the two functions to be looked at over wider numerical values and might give insight into other patterns.

In second place, a study into the relationship between $\phi(n)$ and $\sigma(n)$ in the future may want to look at nonlinear relationships. The models linear relationship seems weak, because the R^2 value is relatively low, less than optimum, which means that the maps will not be a good approximation of their relationship, so nonlinear regression or statistical model can serve as a better model. Thirdly, there might be more number-theoretic variables that could be included in the analysis. In addition to $\phi(n)$ and $\sigma(n)$, one might consider studying the number of different prime factors of n , the greatest prime factor of n , the number of divisors of n , or the prime factorization of n , to name a few functions. This may be useful in deciding what is the most important arithmetic properties in explaining the variation observed.

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