



Reliability Modelling and Performance Evaluation of Active Parallel System with Interactions of Three Failure Types Using Copula Linguistic

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ABSTRACT

This study presents a novel reliability modelling framework for a unit active parallel system subjected to three interacting failure types—minor, medium, and major—using a copula linguistic approach. Traditional reliability models often rely on the assumption of failure independence and binary operational states, which do not accurately reflect the complex failure interactions observed in real-world systems. To address this limitation, we integrate linguistic fuzzy logic with copula theory to capture the dependencies among heterogeneous failure modes while accommodating imprecise failure information. The proposed model accounts for component-level interactions, realistic repair strategies, and escalation pathways among failure types. Analytical expressions are derived for key performance metrics, including availability, mean time to failure (MTTF), and system downtime. Numerical evaluations and sensitivity analyses demonstrate the model's robustness and practical relevance in identifying critical dependence structures and supporting maintenance decision-making. The approach provides a flexible, interpretable, and computationally viable solution for reliability assessment in safety-critical active parallel systems.

CITATION

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INTRODUCTION

The growing complexity of modern engineering systems, particularly those operating in high-reliability environments such as telecommunication, aerospace, and renewable energy systems, necessitates advanced modelling approaches that account for various types of component failures and their interdependencies. Traditional reliability models often assume independence among component failures and categorize system failures into binary states (working or failed). However, real-world systems frequently exhibit multiple failure modes—such as minor, medium, and major failures—whose interactions significantly influence overall system

behavior and resilience. In active parallel systems, where all units operate simultaneously to share the load, understanding the interactions among different failure types becomes essential for accurate reliability and performance evaluation. Two-unit active parallel configurations are widely used due to their structural simplicity and cost-effectiveness, but their reliability analysis becomes intricate when multiple failure modes are considered. Existing studies have extended binary-state reliability models to multi-state systems to better represent such practical scenarios as in Dshalalow (1993) and Chakravorthy et al. (1994).

Modern engineering and industrial systems often demand high reliability, continuous availability, and fault-tolerant performance to meet the expectations of safety, efficiency, and service-level compliance. Among the various architectures employed to achieve these goals, unit active parallel systems—where multiple identical units function simultaneously—have gained prominence due to their inherent redundancy and robustness. These systems are widely used in applications such as telecommunication networks, manufacturing lines, medical monitoring equipment, renewable energy systems, and data centers, where the failure of a single unit should not result in total system shutdown (Rausand & Høyland, 2004). In practical scenarios, components in such systems do not fail independently. The assumption of statistical independence between failure events, which underpins many classical reliability models, often oversimplifies the true nature of complex systems. Multiple types of failures—such as minor, medium, and major—may coexist or influence each other due to shared operating environments, design similarities, or cascading failure effects. For example, a minor fault in a sensor may escalate into a medium-level control fault if undetected, eventually leading to a major system failure. Ignoring the dependency among these failure types can lead to an underestimation of risk and misleading system performance metrics. In these engineering systems, components are often subject to multiple types of failures—such as minor degradations, medium-impact faults, and catastrophic breakdowns—arising from diverse operational and environmental conditions. Traditional reliability modelling approaches tend to assume independence among component failures and rely on precisely defined failure data. However, real-world systems frequently exhibit interdependencies between failure modes due to shared stressors or common-cause events. Moreover, in many practical scenarios, failure data may be incomplete, uncertain, or expressed in linguistic terms by experts or field operators. These limitations hinder accurate reliability assessments and can lead to suboptimal system design or maintenance strategies. There is a pressing need for advanced modelling frameworks that can capture multi-mode failure interactions and incorporate linguistic or imprecise failure information to provide more realistic and robust performance evaluations.

Several studies have addressed reliability modeling of systems subject to two distinct types of failures, often categorized as minor and major failures, to better capture the diverse failure behaviors and their impact on system performance. Dshalalow (1993) investigated retrieval queueing systems considering two failure types, emphasizing their effects on system dynamics. Chakravarthy and Alfa (1994) developed availability models for systems undergoing two failure and repair

modes, providing analytical insights into downtime and operational availability. Wang and Pham (1996) further extended the analysis to systems facing competing degradation and shock-induced failures, highlighting the importance of separate failure process modeling for maintenance optimization.

Repairable systems with partial and total failures were analyzed by Sahoo et al. (2003), who derived key reliability metrics using renewal theory. Li and Pham (2005) contributed a stochastic model distinguishing benign and fatal faults in software systems, demonstrating broader applicability in hardware-software environments. Arivudainambi and Gopalsamy (2007) studied systems with dual failure types alongside multiple repair facilities, revealing the impact of repair prioritization on system availability.

More recent work by Yadavalli and Van Der Linde (2008) focused on two-unit parallel systems with minor and major failures and retrieval repair, employing differential-difference equations to obtain steady-state solutions. Kumar and Sharma (2010) analyzed standby systems with failure-type-dependent repair rates, presenting Laplace transform-based reliability formulations. Tavakkoli-Moghaddam et al. (2011) incorporated degradation modeling alongside failure categorization to better assess system resilience. Bhunia and Roy (2013) developed retrieval queue models for parallel systems with prioritized repairs under minor and major failures, providing practical insights into repair scheduling and system reliability.

As systems increase in complexity, multiple types of failure—such as *minor*, *medium*, and *major*—can co-occur or evolve in an interdependent manner. These interactions are difficult to capture with classical models like exponential or Weibull distributions when used in isolation (Billinton & Allan, 1992). Furthermore, many models consider binary states (working/failed), which overlook the progressive or degraded states that are common in mission-critical and safety-intensive systems (Pham, 2006).

Collectively, these studies demonstrate that incorporating two failure types significantly enhances the fidelity of reliability models, enabling more accurate predictions of system behavior and informing effective maintenance and repair policies.

Copula linguistics—a novel concept combining fuzzy logic and copula theory—offers an expressive modelling language to handle qualitative interactions among multiple failure types, especially in cases where crisp failure boundaries are impractical. This approach enables the incorporation of expert knowledge and imprecise failure categorization into the reliability model, making it particularly useful in domains with uncertainty and vagueness.

The modeling of dependent failure behaviors has received increasing attention in recent years. Among the most

powerful tools for representing such dependencies is the copula function, which provides a mathematically sound and flexible way to couple multiple marginal failure distributions into a joint distribution (Nelsen, 2006). Copulas enable analysts to separate the marginal behavior of each failure type from their interdependence structure, a crucial advantage when dealing with heterogeneous component behaviors.

Traditional copula-based reliability models, however, often emphasize numerical precision at the expense of interpretability, especially when conveying results to non-expert stakeholders or system maintainers. This challenge has motivated the introduction of copula linguistic models, which combine fuzzy linguistic variables and copula theory to enhance both flexibility and human interpretability in capturing the interactions of different failure types (Zhang & Modarres, (2015)). Such an approach is especially relevant in systems where failure types are not easily quantified or where expert judgment plays a role in classification and risk assessment.

Rawal et al. (2021) studied a reliability evaluation of a multicomputer system consisting of n clients functioning under a k -out-of- n : G operational scheme. Their study introduces a Copula-based repair policy to capture the interdependencies among component failures and repairs an aspect frequently neglected in conventional reliability models. This approach enhances the accuracy of reliability assessments in systems where failures are not entirely independent. Singh, Gahlot, and M. (2021) analyze the reliability of a multi-client system with star topology using a Copula-based linguistic approach. By incorporating fuzzy logic and Copula theory, they address uncertainty and dependency in component failures, making the model effective for systems with limited data and complex failure interactions. Raghav, et al. (2021) examined the probabilistic behavior of a complex repairable system composed of two subsystems arranged in series, incorporating a k -out-of- n : G configuration and employing a Copula-based approach to model the repair process and component interdependencies. Ayagi, Singh, and Zhong (2023) evaluated a performance analysis of systems governed by the k -out-of- n : G operational scheme, utilizing Copula functions to model component dependencies and enhance the accuracy of reliability assessments in intricate system environments. Ayagi et. al (2023) developed a performance modeling and estimation framework for serial systems that operate under a k -out-of- n : G configuration, accounting for the possibility of controller failure, thereby enhancing the reliability assessment of such systems.

Fang et al. (2018) used it to model the dependency between components in series and parallel systems, showing improved accuracy in reliability estimation over models that assume independence. Elsaywy and Shebl (2020) investigated the Gumbel copula in the context of k -

out-of- n systems, demonstrating its capacity to express a wide range of dependence structures in standby and active redundancy configurations.

In performance analysis, copula-based methods enable more realistic simulations of system behavior under stress. Jiang and Li (2014) applied the Gumbel-Hougaard copula to study the lifetime distributions of dependent components and showed that system performance metrics like availability and mean time to failure (MTTF) could be significantly misestimated if dependencies are ignored. Furthermore, Yari and Zarei (2012) analyzed the failure dependence in multi-component repairable systems using Archimedean copulas, highlighting the advantages of Gumbel-Hougaard in capturing upper tail dependence under extreme operational conditions.

The application of the Gumbel-Hougaard copula extends beyond static systems. In dynamic reliability models, Khodabin and Ahmadabadi (2013) developed a copula-based stochastic process to model failure interactions over time, incorporating the Gumbel copula to account for tail dependence and aging effects in components.

Despite the theoretical advancements in dependency modeling, existing literature above has not adequately addressed the simultaneous impact of three interacting failure types—minor, medium, and major—within unit active parallel systems. Furthermore, the application of copula linguistic approaches to capture and quantify these interactions remains underexplored. Most available models focus either on binary failure states (working/failed) or assume simplified dependency patterns. As a result, critical failure escalation pathways and nuanced inter-failure dynamics are overlooked, limiting the practical applicability of current models for reliability assessment, especially in safety-critical environments. This study proposes a reliability modelling and performance evaluation framework for a two-unit active parallel system experiencing three distinct types of failures—minor, medium, and major. The study develop a copula linguistic model to capture the interdependence among these failure types, incorporating both the probabilistic structure of copula theory and the linguistic abstraction of fuzzy logic. The model aims to provide a more nuanced and practically relevant reliability analysis for parallel systems operating under complex failure dynamics. The objectives of the study are five folds. First is to develop a reliability modelling framework for a two-unit active parallel system that considers three distinct types of failures: minor, medium, and major. The second is to model the dependency among different failure modes using copula functions, allowing the system behavior to reflect real-world correlations between component failures. Third is to derive and evaluate key performance metrics, such as system reliability, availability, and mean time to failure, under various operational scenarios. The last is to offer insights into the effects of failure

interdependencies and uncertainty on system performance, thereby supporting better decision-making in design, operation, and maintenance across different application domains.

MATERIALS AND METHODS

System Description and Notations

The present paper considers a 2-unit active parallel system in which both units operate simultaneously to process tasks or provide continuous service. The system is designed to maintain functionality as long as at least one of the units is operational. The system is subject to three interacting types of failures, classified as Type 1 (minor failure), Type 2 (intermediate failure), and Type 3 (critical failure), each differing in severity, repair requirements, and impact on system performance. Type 1 Failure (Minor): This failure type represents small, recoverable disruptions such as temporary software glitches, minor hardware faults, or external disturbances. A unit experiencing a Type 1 failure is still functional at a degraded level and may continue operation temporarily, but its performance is reduced. Type 2 Failure (Intermediate): This is a more severe fault than Type 1, possibly involving partial hardware damage or intermittent connectivity loss. A unit undergoing Type 2 failure cannot provide service until it is repaired. The repair process is more involved and takes longer than Type 1 recovery. The failure may also increase the likelihood of cascading failure in the other unit due to load sharing. Type 3 Failure (Critical/Catastrophic): This type results in a complete shutdown of the affected unit. It may arise due to overheating, electrical faults, or irreversible damage. A Type 3 failure requires full replacement or emergency repair and may trigger immediate system-level degradation, especially if both units are affected.

A Type 1 failure in one unit increases the operational stress on the second unit, thereby raising the probability of a Type 2 or Type 3 failure in that unit. Simultaneous minor failures in both units may escalate to higher-severity failures due to mutual load transfer. Type 2 and Type 3 failures in one unit may increase the likelihood of Type 1 or Type 2 failures in the functioning unit due to thermal, electrical, or operational coupling. These interactions are modelled

using conditional failure rates or joint distribution functions, allowing the system to capture dependent failure dynamics rather than assuming independence between units.

Repair and Recovery Strategy

Units experiencing Type 1 or Type 2 failures are sent to a common repair facility on a first-come-first-served basis, with repair times following a general repair distribution. Type 3 failures trigger either emergency repair (copula) or full unit or subsystem or system replacement, which incurs higher costs and downtime. Repaired units by copula are assumed to be as good as new upon restoration.

System Operating Conditions

1. The system is available as long as at least one unit is operational (even if degraded).
2. The system is in a down state only when both units have failed.
3. Reliability and performance metrics, such as availability, MTTF, profit, and MTTF sensitivity, are evaluated based on the transition behavior between states, incorporating failure dependencies and repair processes.

Notations

q : The variable t denotes time across all expressions.

$v_k, k = 1, 2, 3$: Type 1, Type 2, and Type 3 failure rates for the units respectively

$w_0(x), w_0(y), w_0(z)$: Copula repairs addressing the complete failure of units

$R_i(x, q)$: The probability that the system is in state si , undergoing repair, with an elapsed repair time of x at time q , for $i = 1, 4$.

$R_i(y, q)$: The probability that the system is in state si , undergoing repair, with an elapsed repair time of y at time q , for $i = 2, 5$

$R_i(z, q)$: The probability that the system is in state si , undergoing repair, with an elapsed repair time of z at time q , for $i = 3, 6$

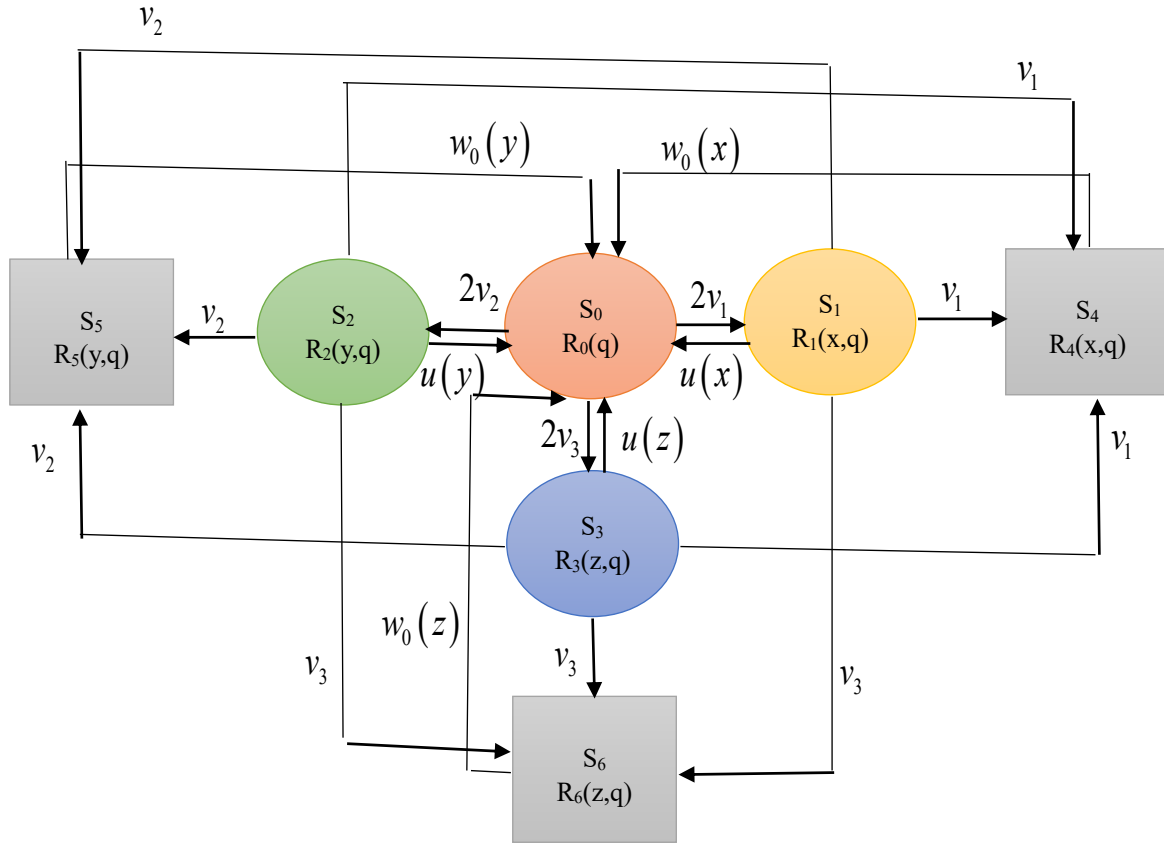


Figure 1: Transition Diagram of Model

Table 1: Depiction of the states of the system

S/N	States	Description
1	S_0	Units are functioning perfectly.
2	S_1	One unit has failed due to Type 1 failure, the second unit is working. The system is up and running in reduced capacity.
3	S_2	One unit has failed due to Type 2 failure, the second unit is working. The system is up and running in reduced capacity.
4	S_3	One has unit failed due to Type 3 failure, the second unit is working. The system is up and running in reduced capacity.
5	S_4	The system is down when Type 1/Type 2/ Type 3 failure induce Type 1 failure on the second unit.
6	S_5	The system is down when Type 1/Type 2/ Type 3 failure induce Type 2 failure on the second unit.
7	S_6	The system is down when Type 1/Type 2/ Type 3 failure induce Type 3 failure on the second unit.

Mathematical Model Preparation and Presentation

The following set of different differential equations are obtained from the system's transition diagram (Figure. 1) by taking into account the chance of disagreements and the impact of relationships as:

$$\left(\frac{\partial}{\partial q} + 2v_1 + 2v_2 + 2v_3\right)R_0(q) = \int_0^\infty u(x)R_1(x,q) dx + \int_0^\infty u(y)R_2(y,q) dy + \int_0^\infty u(z)R_3(z,q) dz + \int_0^\infty w_0(x)R_4(x,q) dx + \int_0^\infty w_0(y)R_5(y,q) dy + \int_0^\infty w_0(z)R_6(z,q) dz \quad (1)$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial x} + v_1 + v_2 + v_3 + \mu(x)\right)R_1(x,q) = 0 \quad (2)$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial y} + v_1 + v_2 + v_3 + \mu(y)\right)R_2(y,q) = 0 \quad (3)$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial z} + v_1 + v_2 + v_3 + \mu(z)\right)R_3(z,q) = 0 \quad (4)$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial x} + w_0(x)\right)R_4(x,q) = 0 \quad (5)$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial y} + w_0(y)\right)R_5(y,q) = 0 \quad (6)$$

$$\left(\frac{\partial}{\partial q} + \frac{\partial}{\partial z} + w_0(z)\right)R_6(z,q) = 0 \quad (7)$$

Boundary conditions

$$R_1(0, q) = 2v_1 R_0(q) \quad (8)$$

$$R_2(0, q) = 2v_2 R_0(q) \quad (9)$$

$$R_3(0, q) = 2v_3 R_0(q) \quad (10)$$

$$R_4(0, q) = 2v_1(v_1 + v_2 + v_3) R_0(q) \quad (11)$$

$$R_5(0, q) = 2v_2(v_1 + v_2 + v_3) R_0(q) \quad (12)$$

$$R_6(0, q) = 2v_3(v_1 + v_2 + v_3) R_0(q) \quad (13)$$

Solution of the model

The Laplace transforms of equations. (1)– (7) can be obtained as follow using the initial condition $P(0) = 1$:

$$\begin{aligned} (s + 2v_1 + 2v_2 + 2v_3)\overline{R_0}(s) &= \int_0^\infty u(x)\overline{R_1}(x, s) dx + \\ &\int_0^\infty u(y)\overline{R_2}(y, s) dy + \int_0^\infty \mu(z)\overline{R_3}(z, s) dz + \\ &\int_0^\infty w_0(x)\overline{R_4}(x, s) dx + \int_0^\infty w_0(x)\overline{R_5}(y, s) dy + \\ &\int_0^\infty w_0(z)\overline{R_6}(z, s) dz \end{aligned} \quad (14)$$

$$\left(s + \frac{\partial}{\partial x} + v_1 + v_2 + v_3 + \mu(x)\right)\overline{R_1}(x, s) = 0 \quad (15)$$

$$\left(s + \frac{\partial}{\partial y} + v_1 + v_2 + v_3 + \mu(y)\right)\overline{R_2}(y, s) = 0 \quad (16)$$

$$\left(s + \frac{\partial}{\partial z} + v_1 + v_2 + v_3 + \mu(z)\right)\overline{R_3}(z, s) = 0 \quad (17)$$

$$\left(s + \frac{\partial}{\partial x} + w_0(x)\right)\overline{R_4}(x, s) = 0 \quad (18)$$

$$\left(s + \frac{\partial}{\partial y} + w_0(y)\right)\overline{R_5}(y, s) = 0 \quad (18)$$

$$\left(s + \frac{\partial}{\partial z} + w_0(z)\right)\overline{R_6}(z, s) = 0 \quad (19)$$

Laplace transforms of Boundary conditions

$$\overline{R_1}(0, s) = 2v_1 \overline{R_0}(s) \quad (20)$$

$$\overline{R_2}(0, s) = 2v_2 \overline{R_0}(s) \quad (21)$$

$$\overline{R_3}(0, s) = 2v_3 \overline{R_0}(s) \quad (22)$$

$$\overline{R_4}(0, s) = 2v_1(v_1 + v_2 + v_3) \overline{R_0}(s) \quad (23)$$

$$\overline{R_5}(0, s) = 2v_2(v_1 + v_2 + v_3) \overline{R_0}(s) \quad (24)$$

$$\overline{R_6}(0, s) = 2v_3(v_1 + v_2 + v_3) \overline{R_0}(s) \quad (25)$$

The above Laplace transforms of the boundary conditions is used to derive equations (26)-(32) through (14)-(19).

$$\overline{R_0}(s) = \frac{1}{M(s)} \quad (26)$$

$$\overline{R_1}(s) = \frac{2v_1}{M(s)} \left\{ \frac{\overline{S_\mu}(s+v_1+v_2+v_3)}{(s+v_1+v_2+v_3)} \right\} \quad (27)$$

$$\overline{R_2}(s) = \frac{2v_2}{M(s)} \left\{ \frac{\overline{S_\mu}(s+v_1+v_2+v_3)}{(s+v_1+v_2+v_3)} \right\} \quad (28)$$

$$\overline{R_3}(s) = \frac{2v_3}{M(s)} \left\{ \frac{\overline{S_\mu}(s+v_1+v_2+v_3)}{(s+v_1+v_2+v_3)} \right\} \quad (29)$$

$$\overline{R_4}(s) = \frac{2v_1(v_1+v_2+v_3)}{M(s)} \left\{ \frac{\overline{S_{w_0}}(s)}{(s)} \right\} \quad (30)$$

$$\overline{R_5}(s) = \frac{2v_2(v_1+v_2+v_3)}{M(s)} \left\{ \frac{\overline{S_{w_0}}(s)}{(s)} \right\} \quad (31)$$

$$\overline{R_6}(s) = \frac{2v_3(v_1+v_2+v_3)}{M(s)} \left\{ \frac{\overline{S_{w_0}}(s)}{(s)} \right\} \quad (32)$$

Where,

$$\begin{aligned} M(s) &= (s + 2v_1 + 2v_2 + 2v_3) - 2v_1 \overline{S_\mu}(s + v_1 + v_2 + v_3) \\ &- 2v_2 \overline{S_\mu}(s + v_1 + v_2 + v_3) - 2v_3 \overline{S_\mu}(s + v_1 + v_2 + v_3) \\ &- 2v_1(v_1 + v_2 + v_3) \overline{S_{w_0}}(s) - 2v_2(v_1 + v_2 + v_3) \overline{S_{w_0}}(s) \\ &- 2v_3(v_1 + v_2 + v_3) \overline{S_{w_0}}(s) \end{aligned}$$

Summing up each Laplace transformation's state change probabilities when the system is operational, we obtain:

$$\overline{R_{up}}(s) = \overline{R_0}(s) + \overline{R_1}(s) + \overline{R_2}(s) + \overline{R_3}(s) \quad (33)$$

$$\begin{aligned} \overline{R_{up}}(s) &= \frac{1}{M(s)} \left\{ 1 + 2v_1 \left\{ \frac{\overline{S_\mu}(s+v_1+v_2+v_3)}{(s+v_1+v_2+v_3)} \right\} + \right. \\ &\left. 2v_2 \left\{ \frac{\overline{S_\mu}(s+v_1+v_2+v_3)}{(s+v_1+v_2+v_3)} \right\} + 2v_3 \left\{ \frac{\overline{S_\mu}(s+v_1+v_2+v_3)}{(s+v_1+v_2+v_3)} \right\} \right\} \end{aligned} \quad (34)$$

$$\overline{R_{down}}(s) = 1 - \overline{R_{up}}(s) \quad (35)$$

System Analysis for Specific (Particular) Cases

This part presents numerical results for each of the considered specific scenarios. For consistency, the following values are set for failure rates; $v_1 = 0.01, v_2 = 0.02, v_3 = 0.03$, throughout.

Availability Analysis

The system availability is given in two different ways to clearly differentiate between the two types of repairs. Viz.,

Availability Analysis for Copula Repair

Here, we set $S_\mu(s) = \tilde{S}_{exp[x^\theta + \{\log \varphi(x)\}^\theta]^{1/\theta}}(s) = \frac{exp[x^\theta + \{\log \varphi(x)\}^\theta]^{1/\theta}}{s + exp[x^\theta + \{\log \varphi(x)\}^\theta]^{1/\theta}}$, $\tilde{S}_h(s) = \frac{\mu}{s + \mu}$, the failure rates are set at various values, such as $\vartheta_1 = 0.01, \vartheta_2 = 0.02, \vartheta_3 = 0.03, \vartheta_4 = 0.04, r = \mu = x = y = z = 1$ and all the repair rates are set equal to 1 i.e. $\mu(x) = \mu(y) = \mu(z) = \omega_0(x) = \omega_0(y) = \omega_0(z) = 1$ in equation (34). Now applying the inverse Laplace transform, we can derive the availability equation for Copula repair as follows:

$$\begin{aligned} \overline{R_{up}}(t) &= 0.1448302930e^{-4.191007466t} - \\ &0.1635110780e^{-2.044912225t} + 1.018680785e^{0.2676196912t} \end{aligned} \quad (36)$$

For different values of time variable $t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, units of time with the aid of equation (36), different values of $P_{up}(t)$ can be obtained as shown in table 2 and figure 2 below.

Table 2: Availability variance with respect to time for Copula Repair

Time	0	1	2	3	4	5	6	7	8	9	10
Availability	1.0000	0.8855	0.8718	0.8566	0.8414	0.8266	0.8119	0.7975	0.7833	0.7693	0.7556

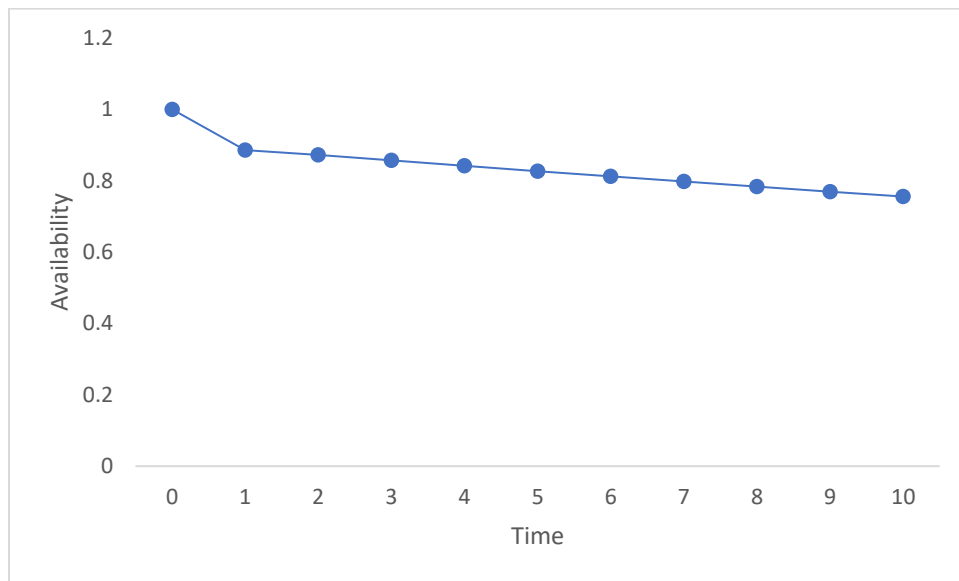


Figure 2: Availability variance with respect to time for Copula Repair

Availability Analysis for General Repair

Letting $\tilde{S}_\mu(s) = \frac{\mu}{s+\mu}$ in equation (34) and differentiating between parameters by assigning various values as $v_1 = 0.01, v_2 = 0.02, v_3 = 0.03$, and $r = 1, \mu = 1$, then taking inverse Laplace transform, we obtain availability equation for general repair as:

$$\overline{R_{up}}(t) = 0.2307491816e^{-1.330386407t} - 0.04532816665e^{-3.122710919t} + 0.8145789852e^{0.2030973259t} \quad (37)$$

For different values of time variable $t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, units of time with the help of equation (37), different values of $R_{up}(t)$ for general repair can be obtained as presented in Table 3 and Figure 3 below.

Table 3: Availability variance with respect to time for General Repair

Time	0	1	2	3	4	5	6	7	8	9	10
Availability	1.0000	0.8867	0.8693	0.8512	0.8333	0.8157	0.7984	0.7814	0.7648	0.7485	0.7325

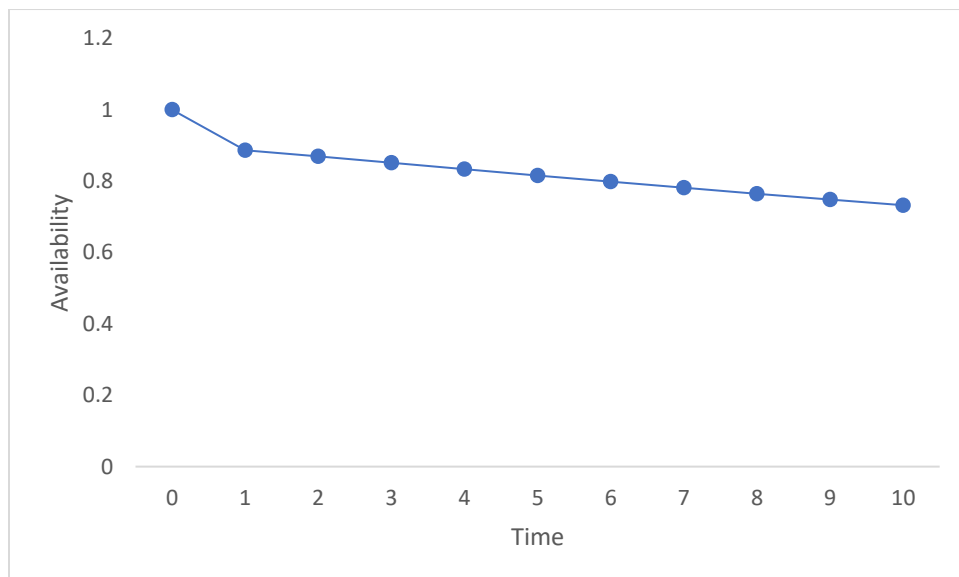


Figure 3: Availability variance with respect to time for General Repair

Reliability Analysis

When the repair is not provided to the failing units or subsystems, the system will most likely operate successfully based on known reliability.

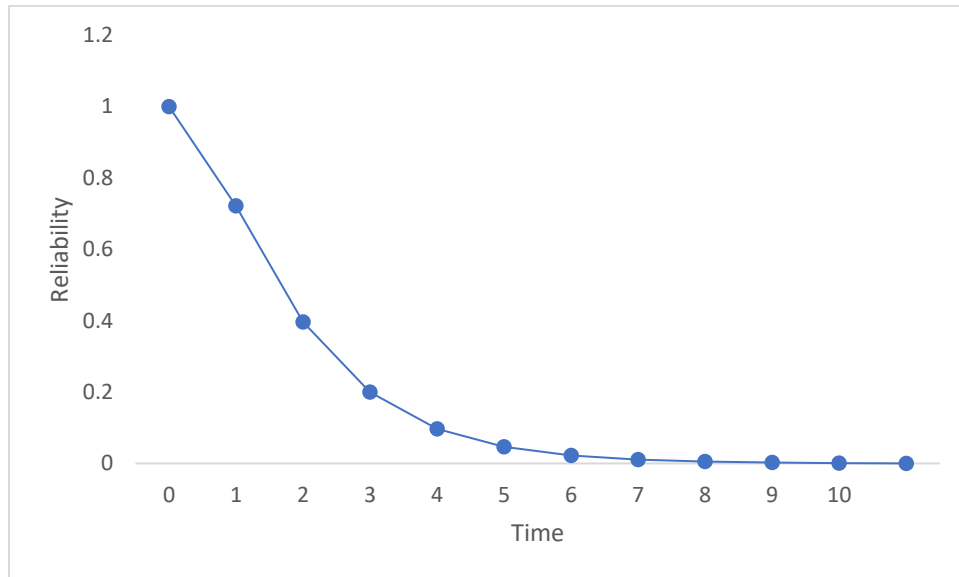
Thus, one may obtain the formula for system reliability by taking all repair rates, $v_1 = 0.01$, $v_2 = 0.02$, $v_3 = 0.03$, from equation (34), then using the inverse Laplace transform.

$$R(t) = 2.00000000e^{-0.7500000000t} - 1.00000000e^{-1.500000000t} \quad (38)$$

For various values of time variable $t = 0.1, 2, 3, \dots, 10$, reliability values have been calculated using the expression in equation (38), as shown in Table 4.

Table 4: Reliability against time t

Time	0	1	2	3	4	5	6	7	8	9	10
Availability	1.0000	0.7216	0.3965	0.1997	0.0971	0.0465	0.0221	0.0105	0.0050	0.0023	0.0011

Figure 4: Reliability against time t **Mean Time To Failure (MTTF) Analysis**

If the repair facility is not available, all of the repairs in equation (34) will be set to zero. When s tends to zero, limit may be used to calculate the MTTF as given below:

$$N(s) = \lim_{s \rightarrow 0} M(s) = 2v_1 + 2v_2 + 2v_3$$

$$MTTF = \lim_{s \rightarrow 0} \overline{R_{up}}(s) = \frac{1}{N(s)} \left\{ 1 + \left\{ \frac{2v_1}{v_1 + v_2 + v_3} \right\} + \left\{ \frac{2v_2}{v_1 + v_2 + v_3} \right\} + \left\{ \frac{2v_3}{v_1 + v_2 + v_3} \right\} \right\} \quad (39)$$

Setting $v_1 = 0.01$, $v_2 = 0.02$, $v_3 = 0.03$, and varying v_1 , v_2 , and v_3 one as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 respectively, the variance of MTTF with respect to failure rates as shown in table 5 and figure 5 can be obtained in the above equation.

Table 5: Computation of MTTF corresponding to the different failure rate values

Failure	MTTF w.r.t v_1	MTTF w.r.t v_2	MTTF w.r.t v_3
0.01	25.0000	30.0000	37.5000
0.02	21.4286	25.0000	30.0000
0.03	18.7500	21.4286	25.0000
0.04	16.6667	18.7500	21.4286
0.05	15.0000	16.6667	18.7500
0.06	13.6364	15.0000	16.6667
0.07	12.5000	13.6364	15.0000
0.08	11.5385	12.5000	13.6364
0.09	10.7143	11.5385	12.5000
0.10	10.0000	10.7143	11.5385

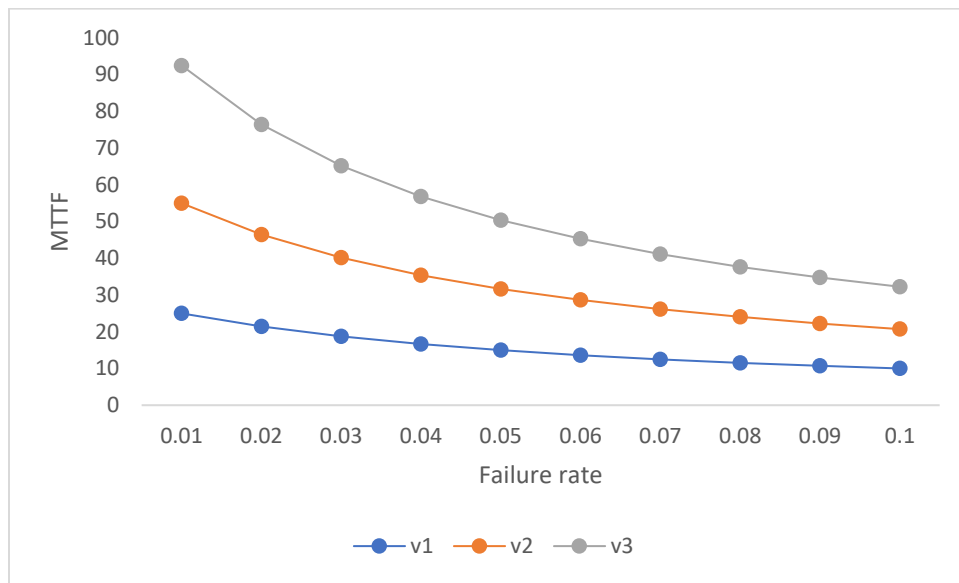


Figure 5: Computation of MTTF corresponding to the different failure rate values

Sensitivity Analysis

The MTTF sensitivity of the system can be obtained by differentiating partially the MTFF with respect to the system failure rates. The MTFF sensitivity is obtained as

shown in table and the corresponding graph in figure by applying the set of parameters as $v_1 = 0.01$, $v_2 = 0.02$ and $v_3 = 0.03$ in the partial differentiation of MTTF.

Table 6: Computation of MTTF corresponding to the different failure rate values

Failure	Sens. w.r.t v_1	Sens. w.r.t v_2	Sens. w.r.t v_3
0.01	-416.6667	-600.0000	-600.0000
0.02	-306.1224	-416.6667	-416.6667
0.03	-234.3750	-306.1224	-306.1224
0.04	-185.1852	-234.3750	-234.3750
0.05	-150.0000	-185.1852	-185.1852
0.06	-123.9669	-150.0000	-150.0000
0.07	-104.1667	-123.9669	-123.9669
0.08	-88.7574	-104.1667	-104.1667
0.09	-76.5306	-88.7574	-88.7574
0.10	-66.6667	-76.5306	-76.5306

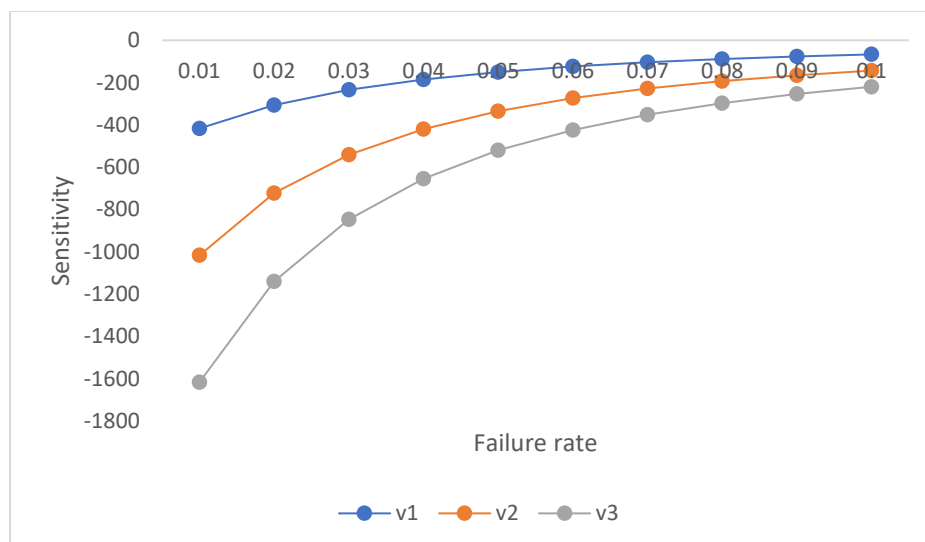


Figure 6: Computation of MTTF corresponding to the different failure rate values

Cost/Benefit Analysis**Cost Benefit Analysis for Copula Repair**

If the service facility is always open/available, then the formula provided below will estimate the expected cost during the interval $[0, t)$.

$$E_p(t) = D_1 \int_0^{\infty} R_{up}(t)dt - D_2 t \quad (40)$$

Where D_1 and D_2 in the interval $[0, t)$ are the revenue generated and service cost per unit time.

Equation (41) can be obtained for the same set of parameters in equation (35).

$$E_p(t) = 0.005321421795e^{-1.172643680t} - 0.001034295514e^{-2.726023675t} + 2731.477195e^{0.0003673545654t} - 2731.481482 - D_2 t \quad (41)$$

Letting $D_1 = 1$ and $D_2 = 0.6, 0.5, 0.4, 0.3, 0.2$ and 0.1 and changing t units of time as $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ respectively, the estimated cost results can be obtained as shown in table 7 and figure 7.

Table 7: Profit computation versus time for Copula Repair

Time(q)	Expected profit $D_2 = 0.1$	Expected profit $D_2 = 0.2$	Expected profit $D_2 = 0.3$	Expected profit $D_2 = 0.4$	Expected profit $D_2 = 0.5$	Expected profit $D_2 = 0.6$
0	0.0000	0.0000	0.00000	0.0000	0.0000	0.0000
1	1.7798	1.5798	1.3798	1.1798	0.9798	0.7798
2	3.5377	3.1377	2.7377	2.3377	1.9377	1.5377
3	5.2661	4.6661	4.0661	3.4661	2.8661	2.2661
4	6.9638	6.1638	5.3638	4.5638	3.7638	2.9638
5	8.6320	7.6320	6.6320	5.6320	4.6320	3.6320
6	10.2702	9.0702	7.8702	6.6702	5.4702	4.2702
7	11.8795	10.4795	9.0795	7.6795	6.2795	4.8795
8	13.4606	11.8606	10.2606	8.6606	7.0606	5.4606
9	15.0128	13.2128	11.4128	9.6128	7.8128	6.0128
10	16.5377	14.5377	12.5377	10.5377	8.5377	6.5377

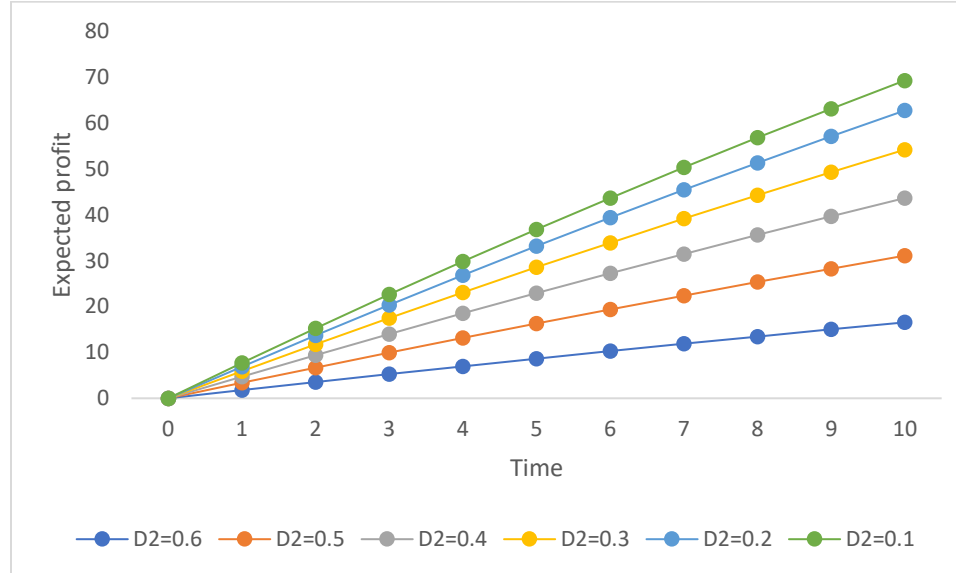


Figure 7: Profit computation versus time for Copula Repair

Cost Benefit Analysis for General Repair

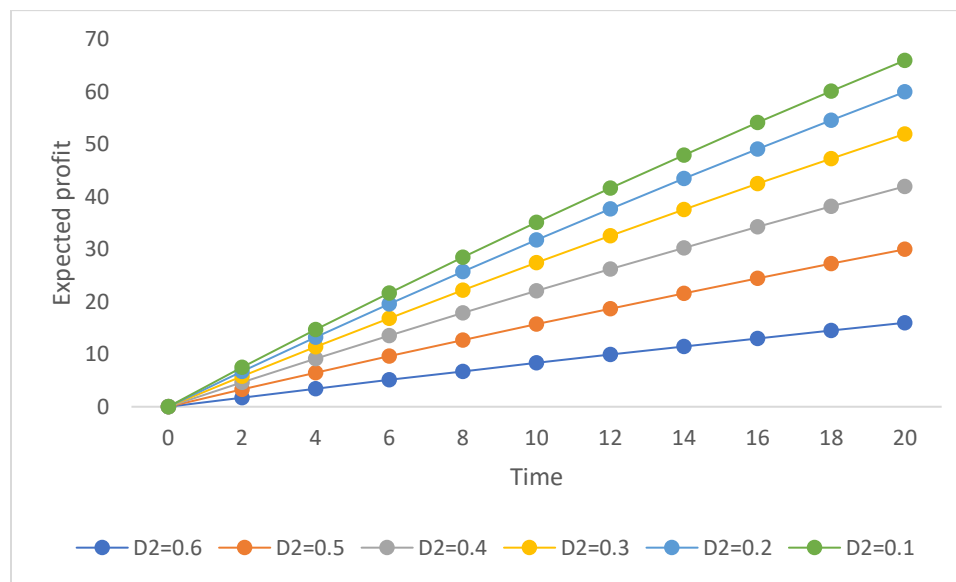
The expected profit/cost function for general repairs using the parameters of equation (36) is obtained by:

$$E_p(t) = 0.001505475341e^{-1.177938943t} - 0.002442644281e^{-0.7500000000t} + 2731.482418e^{0.0003658543723t} - 2731.4814810 - D_2 t \quad (42)$$

Table 8 and Figure 8 are obtained by applying the inverse Laplace transform to equation (42) with $D_1 = 1$ and $D_2 = 0.06, 0.05, 0.04, 0.03, 0.02$, respectively, and using various values of the time variable, such as $t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$.

Table 8: Profit computation versus time for General Repair

Time(q)	Expected profit $D_2 = 0.1$	Expected profit $D_2 = 0.2$	Expected profit $D_2 = 0.3$	Expected profit $D_2 = 0.4$	Expected profit $D_2 = 0.5$	Expected profit $D_2 = 0.6$
0	0.0000	0.0000	0.00000	0.0000	0.0000	0.0000
2	1.7496	1.5496	1.3496	1.1496	0.9496	0.7496
4	3.4475	3.0475	2.6475	2.2475	1.8475	1.4475
6	5.1140	4.5140	3.9140	3.3140	2.7140	2.1140
8	6.7513	5.9513	5.1513	4.3513	3.5513	2.7513
10	8.3607	7.3607	6.3607	5.3607	4.3607	3.3607
12	9.9425	8.7425	7.5425	6.3425	5.1425	3.9425
14	11.4970	10.0970	8.6970	7.2970	5.8970	4.4970
16	13.0246	11.4246	9.8246	8.2246	6.6246	5.0246
18	14.5258	12.7258	10.9258	9.1258	7.3258	5.5258
20	16.0010	14.0010	12.0010	10.0010	8.0010	6.0010

**Figure 8: Profit computation versus time for General Repair**

Discussion of Results

The data presented in Table 2 and Figure 2, as well as in Table 3 and Figure 3, clearly reveal a gradual decline in system availability over time under both copula-based and general repair policies. This downward trend indicates a decreasing likelihood of the system remaining operational as time progresses, reflecting a deterioration in both system reliability and performance. The observed decline may be attributed to the cumulative effects of component aging and wear, the presence of correlated failures that are not fully mitigated by the copula-based repair policy, and potential limitations in repair capacity or efficiency. Although the copula-based repair strategy provides a more advanced framework for modeling dependency among failures and repairs, it does not completely prevent the gradual loss of availability. This underscores the necessity for proactive maintenance interventions, increased repair capabilities, or system upgrades to sustain reliability and ensure long-term performance.

The data presented in Table 4 and Figure 4 clearly reveal a rapid decline in system reliability over time when the system is subjected to three types of failures—namely, minor, medium, and major failures. This downward trend reflects a diminishing likelihood of the system remaining operational as time progresses, signifying a notable deterioration in both reliability and overall performance. The presence of multiple failure severities introduces additional complexity, as minor failures may accumulate and evolve into more severe conditions, medium failures may impair partial system functionality, and major failures often result in total system shutdown. The combined effect of these failure types accelerates system degradation and places increased demand on repair mechanisms. If the repair strategy is not tailored to address each failure type appropriately, system recovery becomes inefficient, leading to longer downtimes and lower availability. This highlights the importance of implementing differentiated repair policies, redundancy schemes, or predictive

maintenance frameworks to manage failure severity, improve recovery efficiency, and sustain system reliability and performance over time.

Table 5 and Figure 5 illustrate the impact of the three distinct failure rates v_1 , v_2 , and v_3 on the Mean Time to Failure (MTTF) of the system. These failure rates correspond to three different types of failures that the system may experience: minor, medium, and major. As the failure rates increase within the range of 0.01 to 0.10, the MTTF consistently decreases, indicating a reduction in system reliability and an increased likelihood of earlier system breakdown. Among the three failure rates, MTTF is observed to be highest concerning v_3 across the range considered. This suggests that, relative to the other failure modes, the failure associated with v_3 has a less immediate impact on the system's expected time to failure, possibly due to it occurring less frequently or being less critical in the system's architecture. Conversely, the failure rates v_1 and v_2 are more sensitive to increases, implying that the failure modes they represent contribute more significantly to the decline in system reliability.

Table 6 and Figure 6 illustrate the sensitivity of the Mean Time to Failure (MTTF) to variations in the failure rates v_1 , v_2 , and v_3 . The results reveal that MTTF sensitivity increases as the failure rates rise, indicating that these failure parameters highly influence the system's reliability. Failure rate v_1 exhibits the greatest sensitivity among the three, underscoring its dominant impact on system performance degradation.

Tables 7 and 8, along with Figures 7 and 8, present the time-dependent profit analysis under varying service costs $D_2=0.1, 0.2, 0.3, 0.4, 0.5, 0.6$ for both the copula-based and general repair models. These results highlight the intricate relationship between repair strategy, service cost, and profitability over time. As time progresses, system profit consistently increases across all service cost scenarios, indicating improved productivity and operational efficiency due to sustained system performance. Notably, the lowest service cost ($D_2=0.1$) yields the highest profit in both models, emphasizing the economic advantage of minimizing service costs in system maintenance strategies. Furthermore, the copula-based repair model outperforms the general repair model in terms of profit generation across all cost levels. This demonstrates that accounting for dependency structures among component failures (via copula modeling) not only enhances system reliability and performance but also supports better decision-making for cost optimization and long-term system profitability.

CONCLUSION

This study presents a comprehensive reliability and performance analysis of a two-unit active parallel system subjected to three interacting failure types—Type 1, Type 2, and Type 3. By employing Laplace transform techniques

alongside the method of supplementary variables, we developed closed-form expressions for key system effectiveness metrics, including system reliability, steady-state availability, mean time to failure (MTTF), MTTF sensitivity, and long-run profit. The model captures the complex dynamics of failure interactions, offering a realistic and insightful representation of multi-failure behavior in mission-critical systems. The novelty of this study lies in its formulation of a multi-failure-type active parallel retrial system that incorporates dependent failure interactions, which are often overlooked in classical reliability models. By integrating MTTF sensitivity and economic performance (profit), the model provides a multi-dimensional view of system performance. Moreover, the comparative analysis between different failure types and cost structures introduces a new framework for linking failure behavior to operational and financial outcomes. The primary contributions of this work are threefold: (1) the development of a generalized analytical framework for evaluating reliability and performance under interacting failure types; (2) the introduction of sensitivity analysis concerning failure rates to guide targeted reliability improvement strategies; and (3) the incorporation of a profit-based metric for assessing long-term system productivity and cost-effectiveness. These contributions provide a robust decision-support tool for reliability engineers, system designers, and operations managers.

From a performance optimization standpoint, the model demonstrates how service cost structures and failure dependencies influence system uptime, profitability, and overall productivity. The results underscore the critical importance of minimizing high-impact failure rates and optimizing repair strategies to achieve cost-effective and resilient system performance. Looking ahead, this research opens several promising directions. Future work may extend the model to incorporate higher-order failure dependencies using advanced copula-based techniques, heterogeneous repair policies, or machine learning-assisted predictive maintenance. Additional extensions may involve modelling systems with time-varying failure rates, renewable or smart energy components, or limited maintenance resources. Investigating the interplay between reliability, resilience, and sustainability—particularly in energy, agriculture, or defense sectors—would also broaden the applicability and societal relevance of the framework.

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